Matlab® Software as a Teaching Tool for the Design of Second-Order Passive Filters

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Abstract

The design of passive filters, mainly second-order filters, may be difficult to understand by Electrical Engineering students. This may be due to the use of complex equations and damping oscillations. This design process can be simplified by using Laplace transforms. The current generation of students, whose background is more visual than in the past, seems to enjoy, and learn abstract concepts better, when they are involved in classes that include laboratory experiments (Amieva & Zapata, 2005). This paper presents the methodology used to simplify the filter design process. Three different types of filters are analyzed and their frequency response is visualized using Matlab®.

Introduction

One of the main problems in the area of electronic communications and signal processing is the presence of unwanted signals that exist in the environment such as electromagnetic interference produced by electric and electronic devices. These extraneous signals introduce unwanted noise that when mixed with the true signal, produces unwanted results.

A filter is a circuit designed to accept signals with desired frequencies and reject or attenuate others (Alexander & Sadiku, 2004). A filter is passive when it uses components like a resistor (R), in combination with an inductor (L) and/or a capacitor (C), without using an amplifier. Filters rely on the fact that the inductive (Z_L) and capacitive impedance (Z_C) change as a function of frequency.

The key to simplify the analysis of the frequency response of these filters is to follow three simple steps:

1. Transform the filter circuit elements from the time (t) domain to the frequency (s) domain using Laplace transforms.
2. Use the voltage divider rule to find the output voltage, and represent the equation as a transfer function H(s), the ratio of the output to the input voltage.
3. Use Matlab® software to display the frequency response.

Laplace Transform of the Transfer Functions

In order to analyze an RLC filter it is necessary to know the relationship between the voltage difference across a given circuit element and the corresponding current flow (Zornesky & Maybar, 2000).

The voltage across a resistor is a function of time given by $V(t) = R \cdot i(t)$, with Laplace transform $\mathcal{L}\left\{R \cdot i(t)\right\} = \int_0^{\infty} R \cdot i(t) \cdot e^{-s \cdot t} dt = R\int_0^{\infty} i(t) \cdot e^{-s \cdot t} dt = R \cdot I(s)$,

where $I(s) = \int_0^{\infty} i(t) \cdot e^{-s \cdot t} dt$ is the Laplace transform of the current $i(t)$.

The voltage across an inductor is given by $V(t) = L \cdot \frac{di(t)}{dt}$, with Laplace transform

$$\mathcal{L}\left\{L \cdot \frac{di(t)}{dt}\right\} = \int_0^{\infty} L \cdot \frac{di(t)}{dt} \cdot e^{-s \cdot t} dt = L\left(i(t) \cdot e^{-s \cdot t}\Big|_0^{\infty} + s\int_0^{\infty} i(t) \cdot e^{-s \cdot t} dt \right)$$

$$= L\left(i(0) + s \cdot I(s)\right) = s \cdot L \cdot I(s) \text{ since } i(0) = 0.$$

The voltage across a capacitor is given by $V(t) = \frac{1}{C}\int i(t)dt$ with Laplace transform

$$\mathcal{L}\left\{\frac{1}{C}\int i(t)dt\right\} = \int_0^{\infty} \frac{1}{C}\int i(t)dt \cdot e^{-s \cdot t} dt = \frac{1}{C}\left(-\frac{1}{s} \cdot e^{-s \cdot t}\int i(t)dt\Big|_0^{\infty} + \frac{1}{s}\int_0^{\infty} i(t) \cdot e^{-s \cdot t} dt \right)$$

$$= \frac{1}{C} \cdot \frac{1}{s} \cdot I(s).$$

Table I summarizes these transformations.

Voltage Divider

The voltage divider rule is used to find the output voltage (V_0) equation for every filter. For the low pass filter, the output voltage is the voltage across the capacitor (V_C) given by the expression,

$V_C = V_i\left(\frac{Z_C}{Z_R + Z_L + Z_C}\right) = \frac{V_i\left(\frac{1}{sC}\right)}{R + sL + \frac{1}{sC}} = \frac{V_i\left(\frac{1}{sC}\right)}{\frac{sCR + s^2LC + 1}{sC}} = \frac{V_i}{sCR + s^2LC + 1}$, with resulting transfer function

$H(s) = \frac{V_C}{V_i} = \frac{1}{sCR + s^2LC + 1} = \frac{\frac{1}{LC}}{s^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC}}$, a second-order transfer function.

For the high pass filter, the output voltage is the voltage across the inductor (V_L) given by

$V_L = V_i\left(\frac{Z_L}{Z_R + Z_L + Z_C}\right) = \frac{V_i(sL)}{R + sL + \frac{1}{sC}} = \frac{V_i sL}{\frac{sCR + s^2LC + 1}{sC}} = \frac{V_i\left(s^2LC\right)}{sCR + s^2LC + 1}$, resulting in the transfer function

$H(s) = \frac{V_L}{V_i} = \frac{s^2LC}{sCR + s^2LC + 1} = \frac{s^2}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$

Lastly, for the band pass filter, the output voltage is the voltage across the resistor, given by $V_R = V_i\left(\frac{Z_R}{Z_R + Z_L + Z_C}\right) = \frac{V_i(R)}{R + sL + \frac{1}{sC}} = \frac{V_i sRC}{sCR + s^2LC + 1}$, which yields the transfer function $H(s) = \frac{V_R}{V_i} = \frac{sRC}{sCR + s^2LC + 1} = \frac{s\left(\frac{R}{L}\right)}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$

Generic Transfer Function of Second Order Filters

As mentioned previously, the response of every electrical circuit varies as a function of operation frequency. One way to express these changes in behavior is to construct transfer functions.

For the three filters shown in Figure I, the transfer function H(s) takes the general form

$$H(s) = \frac{Vo}{Vi} = \frac{b_m S^m + b_{m+1} S^{m+1} + \ldots\ldots.b_1 s + b0}{a_n s^n + a_{n-1} s^{n-1} + a_1 s + a_0}$$

This frequency function, using Matlab® software, takes the general form: *s=freqs(num,den,range)*, whose use is detailed in the next section.

Use of Matlab® as a Mathematical Analysis Tool

Matlab® software is used extensively in the engineering field (Etter 1997), due to its excellent plotting capabilities. In order to use the Matlab® function specified previously it is necessary to define the numerator and denominator of the transfer function of the system under consideration. Figures II, III and IV, show the Matlab® programs and the graphs of the resulting frequency response for the low pass, high pass, and band pass filters, respectively.

Figure II (Low Pass Filter) shows that those frequencies up to 10^2 Hz pass readily. However, as the frequency approaches 10^3 Hz only 60% passes.

Figure III (High Pass Filter) shows that those frequencies above 10^4 Hz pass completely. The band pass filter shown in figure IV is a combination of the low and high pass filters as only the frequencies around 10^3 Hz pass, while rejecting lower or higher frequencies.

Conclusion

The use of Matlab® software has proven to be helpful to our electrical engineering students. This paper helps to illustrate one instance in which the use of Matlab® software functions has been useful to our students to understand the process of design of RLC filters.

Table I
Laplace transform representation of passive circuit elements.

Component	Symbol	Time Domain	Laplace Domain	Impedance $Z(s)=\frac{V(s)}{I(s)}$
Resistor		$V(t)=R\cdot i(t)$	$V(s)=R\cdot I(s)$	$Z_R(s)=R$
Inductor		$V(t)=L\cdot\frac{di(t)}{dt}$	$V(s)=s\cdot L\cdot I(s)$	$Z_L(s)=s\cdot L$
Capacitor		$V(t)=\frac{1}{C}\cdot\int i(t)dt$	$V(s)=\frac{1}{s\cdot C}\cdot I(s)$	$Z_C(s)=\frac{1}{sC}$

Figure I
Low Pass Filter, High Pass Filter, and Band Pass Filter

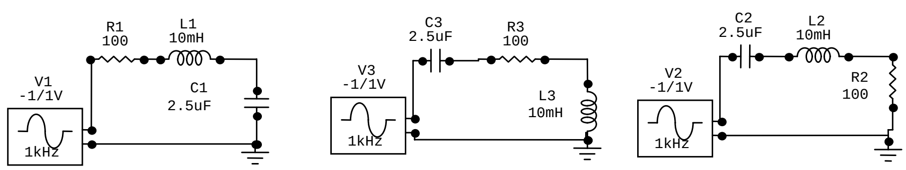


Figure II
Low Pass Filter

```
% Low Pass Filter
num=[39.4e6];
den=[1,10e3,39.4e6];
w=logspace (2,5);
h=freqs(num,den,w);
f=w/(2*pi);
mag=abs(h);
semilogx(f,mag);
title('Two Pole LPF Frequency Response'
xlabel('Frequency, Hz');
ylabel('Magnitude');
Grid
```

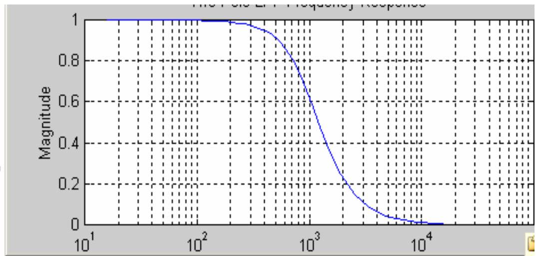


Figure III
High Pass Filter

```
% High Pass Filter
num=[1 0 0];
den=[1,10e3,39.6e6];
w=logspace (2,6);
h=freqs(num,den,w);
f=w/(2*pi);
mag=abs(h);
semilogx(f,mag);
title('Two Pole HPF Frequency Response'
xlabel('Frequency, Hz');
ylabel('Magnitude');
Grid
```

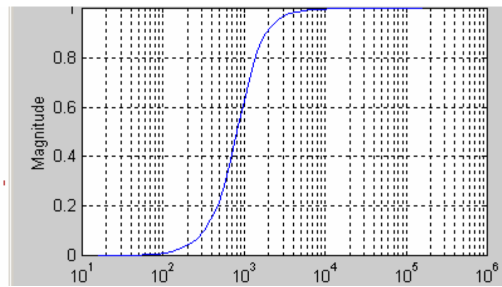
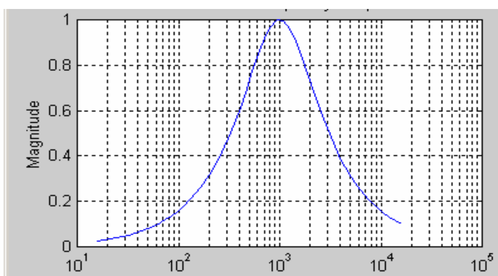


Figure IV
Band Pass Filter

```
%Band Pass Filter
num=[10e3 0];
den=[1,10e3,39.4e6];
w=logspace (2,5);
h=freqs(num,den,w);
f=w/(2*pi);
mag=abs(h);
semilogx(f,mag);
title('Two Pole BPF Frequency Response'
xlabel('Frequency, Hz');
ylabel('Magnitude');
Grid
```



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