Spaced Reinforcement: An Effective Approach to Enhance the Achievement in Plane Geometry

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Abstract

The purpose of the study was to design and investigate the efficacy of spaced reinforcement versus massed reinforcement on the learning and retention of plane geometry by high school students. The design consisted of one control and two experimental groups. The daily reinforcement for the massed treatment was limited to the given day's lessons. Alternatively, the daily reinforcement for the spaced treatment consisted of the given day's lesson and a variety of previously taught material according to a pre-designed schedule. The time on task was the same for all groups during the treatment. The pretest indicated that the students were equivalent initially. Differences of achievement were compared using an analysis of covariance on the posttest. Test statistic indicated that the spaced treatment resulted in a measurably greater level of achievement compared to the massed treatment.

Statement of the problem

The contributions of geometry to the development and advancement of the other branches of mathematics, the natural sciences, and technology, are widely recognized by scholars across the disciplines. Yet we, as mathematics and science educators, too often witness the disappointment and struggles of students in learning, recalling, and applying geometric principles in their subsequent coursework. Further, we observe that most students in high school geometry classes perform at an acceptable level as they study each concept when it is presented, yet many of these same students do not perform well on comprehensive final exams in geometry. The present paper describes an experimental study in which we examined the relative effectiveness of two alternative modes of instructional reinforcement in high school geometry classes: spaced (or distributed) reinforcement and massed reinforcement. We operationally define the two current methods of reinforcing student learning through homework assignments as follows:

- Massed Reinforcement is the traditional method of drilling, in which the newly-taught concepts or skills, shortly after they are presented to the learner, are applied many times within a concentrated period of time until mastery is (apparently) achieved.

- Spaced Reinforcement (also referred to as *distributed reinforcement)* is a strategy in which the newly-taught concepts or skills are learned simultaneously with the reinforcement of a variety of previously-taught concepts and skills over an extended period of time, usually five or more non-consecutive class sessions. For experimental purposes (and ideally in practice as well), this method follows a designed sequence of a daily percentage composition of reinforcements of previous student learning.

Our study suggests that the spaced reinforcement strategy is significantly superior in engendering students' meaningful long-term learning.

Perspectives or theoretical framework

As mathematics educators we observe that students struggle to apply a prerequisite understanding of geometry in other courses such as trigonometry, advanced mathematics, and physics. Furthermore, many students fail to recognize the applications of such concepts as area, volume, and geometric summation of vectors. There are various signs that suggest a pattern of learners' difficulties in retaining their geometric skills even after they have successfully passed geometry. Moreover, for many students who plan to enter the work force upon high school graduation, basic geometrical skills are essential in trades such as machining, manufacturing, construction, pipe fitting, carpentry, and heating and air conditioning. The issue of preparing students to succeed in such a variety of endeavors places serious responsibilities on the geometry teacher.

Because no issue in education ever arises on a blank slate, we began by looking at the historical antecedents to the current situation. In 1894, the *Report of the Committee of Ten*, recommended that geometry be taught to all students. NCTM, (1933) stated "[The report indicates that] it is the belief of [the] conference that the course here suggested [geometry], if skillfully taught, will not only be of great educational value to all children, but will also be a most desirable preparation for later mathematical work." We went on to examine a sampling of geometry textbooks published between 1854 and 1999, focusing our attention on instructional design, sequencing of topics, and techniques of reinforcement. In doing so, we qualitatively observed a gradual emergence, most notably after the mid-20th century, of a trend toward spaced reinforcement in the review sections at the ends of the chapters in geometry texts. This, of course, did not prove that spaced reinforcement was more effective pedagogically, nor did it guarantee that teachers actually utilize their textbook's spaced reinforcement review strategy. Our obvious questions were these: Does spaced reinforcement work? Is there evidence that this apparent trend toward spaced reinforcement is justified? And, if so, is spaced reinforcement effective enough that geometry teachers should adopt it as a standard pedagogical practice?

Methods and procedures

Our six-week experimental study involved 169 tenth- and eleventh-grade geometry students at a low-achieving public high school in Baton Rouge, Louisiana. Participants were randomly assigned to three groups (two separate class sections per group): sixty-one students were in the massed reinforcement group, fifty-nine in the spaced reinforcement group, and forty-nine in a control group. The massed and spaced reinforcement groups received the identical geometry instruction from the standard curriculum, while students in the control group were not enrolled in a geometry course and received no treatment at all. The control group was enrolled in business mathematics. The control group enabled us to define a baseline against which the two treatments were compared. The control group was significant because the results derived yielded an absolute difference rather than a relative one. All of the instruction was done by teachers other than the investigators. A team of two certified teachers with a combined

41 years experience teaching geometry delivered the instruction to the massed reinforcement group. A second team of two certified teachers with a combined 39 years experience teaching geometry was in charge of instruction for the spaced reinforcement group. One teacher taught the control group. For the spaced reinforcement, we used a modification of the quantity of reinforcements proposed by Laing (1970), Urwiller (1971), Camp (1973), and Weaver (1970). Table I illustrates this procedure for topic A as an example.

Table I
The Spaced Schedule for Reinforcement on Topic A

Day	% of the Homework	Day	% of the Homework	Day	% of the Homework	Day	% of the Homework
1	30	7	15	13		19	
2	20	8		14		20	5
3		9		15	5	21	
4	15	10		16		22	
5		11	10	17		23	
6		12		18		24	

The massed reinforcement group, on the other hand, was assigned the standard homework, which always focused on the current topic(s). We confirmed that the total amount of homework done by both student groups was the same, and that the time-on-task of both groups was virtually identical.

For the evaluation instruments, we used two standardized tests for which national norms were available. One test was the National Achievement Tests: Plane Geometry. The other test was, Van Hiele Geometry Test, an instrument developed in 1980 by Usiskin and the CDASSG Project at the University of Chicago. The Van Hiele Geometry Test, which focuses not only on content knowledge of geometry but on the sophistication of the student's conceptual level of understanding those concepts. Both of these instruments were administered to all three experimental groups as a pretest and as a post-test. Additionally, both tests were again administered to the two treatment groups (but not to the control group) six weeks later as a retention test.

Results and Conclusions

As shown in Tables II and III, we found no significant difference in the students' (massed reinforcement, spaced reinforcement, and control group) prior knowledge of geometry. Further, prior to the experimental treatment they all performed, on average, at the same conceptual level 1 of the Van Heile 5-level scale.

Table IV and V summarizes the posttest results. The spaced reinforcement group showed more gain than the massed reinforcement group, as confirmed ($\alpha = 0.05$) by both an ANCOVA analysis and the Tukey/Kramer test. Further, the students in the spaced reinforcement group advanced to level 3 while the students in the massed reinforcement group advanced to level 2 of the Van Heile scale of conceptual understanding. The control group, as expected, showed no gain at all between the pretests and posttests.

Finally, in Table VI, we see that six weeks after the experiment, the students who had participated in the spaced reinforcement treatment retained significantly more knowledge ($\alpha = 0.05$) of geometry than those who received the massed reinforcement treatment. We eliminated the control group after the posttest.

Table II
The Groups' Mean Scores for the Pretest

Group	Control	Massed	Spaced
Number Students	49	61	59
Standard Deviation	1.25	1.64	1.88
Range	9.00	11.0	10.0
Mean score	9.77	10.45	10.51

National Norm = 24.34

Table III
ANOVA Assessing Performance Differences among Groups on the Pretest

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F prob.
BG	2	17.5051	8.7525	1.2696	.2837
WG	166	1144.4239	6.8941		
Total	168	1161.9290			

BG = Between Groups WG = Within Groups

Table IV
The Groups' Mean Scores for the Posttest

Group	Control	Massed	Spaced
Number of Students	49	61	59
Standard Deviation	1.26	1.53	1.26
Range	10	9	7
Mean Score	9.16	20.41	25.35

National Norm = 24.34

Table V
ANCOVA Assessing Performance Differences among Groups on the Posttest

SV	Sum of Squares	DF	Mean Squares	F	Sig. of F
Covariates	34.059	1	34.059	10.547	.001
MEG	1733.645	2	866.823	268.44	.000
Explained	1870.798	3	623.599	193.11	.000
Residual	532.800	165	3.229		
Total	2403.598	168	14.307		

SV = Source of Variance MEG = Main Effect Groups

Table VI
The Groups' Mean Scores for the Retention Test

Group	Massed Treatment	Spaced Treatment
Number of Students	61	59
Standard Deviation	1.52	1.45
Range	13.0	12.0
Mean Score	19.63	24.25

Test statistic indicated that the spaced reinforcement treatment resulted in a significantly greater level of achievement in plane geometry compared to the massed reinforcement treatment. On this basis, we must conclude that there is a substantive justification for the shift in emphasis on the part of authors of geometry textbooks toward including spaced reinforcement as a part of their instructional designs.

Implications for Practice

Our most dramatic finding is that sixty-one students in a low-performing urban high school outperformed (on average) the national norm on a standardized test in geometry, and their learning was retained. Our study strongly suggests that in general:

- Reliance on the classic method (the massed method) of reinforcement is not optimal to enhancing student performance or their conceptual understanding of geometry.
- There exists a substantive rationale for the current trend of authors such as Saxon (1985) and Usiskin et al. (1998) for incorporating spaced strategies in the instructional design of their textbooks.

Given the evidence that spaced (distributed) reinforcement is significantly more effective than traditional massed reinforcement as an instructional strategy, we strongly urge textbook adoption committees to give preference to those prospective texts that incorporate this feature. Further, to those of our colleagues who may not already regularly draw upon a spaced reinforcement strategy in devising student assignments, we strongly recommend that they consider doing so. Without spending any additional time on-task, students clearly do acquire and retain a more extensive understanding of geometry at a deeper conceptual level when each of their assignments includes distributed material from earlier lessons.

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