Teaching a Mathematics Course for Future Teachers On-Line: Opportunities and Challenges

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Abstract

This paper discusses the opportunities and challenges of an on-line environment for teaching non-traditional math courses for Liberal Studies students. The courses are intended to deepen future teachers' knowledge of elementary school mathematics and provide them with a background in how children learn the particular mathematics topics. The courses primarily use cooperative learning, manipulatives, and constructivist activities as teaching strategies. The teacher does not show students how to solve problems. Students develop and share their own ideas with their classmates. This type of instruction cannot easily be conducted in an on-line environment. However, we were required to make the courses available on-line, and thus, adapted them to this environment, trying to maintain our teaching and learning philosophy. In doing so, we have found that some of our fears have been realized and others have not. In fact, the on-line environment has provided some unexpected opportunities for developing a community of learners in our classes. This paper describes the on-line courses including the structures we use to promote collaboration and inquiry and it discusses challenges we still face in trying to provide the same type of learning environment that the live classes provide.

Many U.S. elementary teachers lack a deep understanding of the mathematics they teach, and prospective elementary teachers often fear and dislike math (Hembree, 1990; Leinhardt & Smith, 1985; Mewborn, 2000). Teacher preparation is key to improving elementary teachers' attitudes towards and understanding of the mathematics they teach (Ma, 1999). At CSUMB, liberal studies students are required to take a lower division math course and two upper division math courses that were developed specifically for students who plan to become elementary school teachers. Recently we have received pressure to offer the two courses on-line (in addition to "live") for distance-learning students and for those whose schedule cannot accommodate coming to campus. Because the courses use innovative teaching approaches (such as cooperative learning, manipulatives, constructivist activities), the adaptation to on-line was not simple. This paper describes the courses, the adaptations to the on-line environment, and the challenges and benefits of the on-line environment.

Description of the Courses and Pedagogical Theories behind Them

Math 308 and 309 (Elementary Math from an Advanced Perspective) is a required upper division two-course sequence for liberal studies students—students who intend to become elementary school teachers. The purpose is for students to gain an in-depth understanding of some key concepts in elementary mathematics and to develop insight into how these concepts, mathematical content, and processes are learned. Both

308 and 309 are inquiry-based and use manipulatives when possible to model simple mathematical situations.

The general mathematical content of the courses includes**:** Historical references and the development of modern number system notation; base ten place value concepts and representations (whole numbers and decimal numbers); meanings, models, and algorithms for operations (Addition, Subtraction, Multiplication, and Division); number theory; fractional numbers (meanings, models, conversion between different notations, operations); algebraic thinking, statistics, probability, geometry, measurement. The courses focus on making sense of mathematics—problem solving, mathematical inquiry and children's cognition.

Journal of Mathematical Sciences & Mathematics Education

The pedagogy of the courses is based on both cognitive and social constructivism—that is, the belief that people construct their own knowledge and understanding, and they do so in social contexts the dynamics of which significantly influence individual learning. Our cognitive constructivist views stem from compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. According to the national report, *Everybody Counts*, "All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them. Students do not learn simply a subset of what they have been shown. Instead, they use new information to modify their prior beliefs. As a consequence, each student's knowledge of mathematics is uniquely personal" (NRC, 1989). This constructivist view is embodied in the NCTM Standards, which equate learning mathematics with doing mathematics. Rote and passive learning of mathematical facts and procedures is not adequate—understanding, explanation, and prediction are crucial elements of learning and doing mathematics. Therefore, students in these courses need to draw inferences, interpret models, estimate results, suggest alternatives, and make reasonable testable guesses, as they apply mathematical methods to solutions of mathematical (and often real world) problems.

Instead of basing courses on rote computation, the NCTM standards recommend activities that convey a deeper conceptual understanding of mathematics. To effectively facilitate this type of learning, teachers must understand clearly the underlying concepts in the math activities themselves and make sure students understand them. Math 308 and 309 use these types of activities both to deepen students' knowledge of the math and to model the use of such activities.

Some ideas from social constructivism that have been most important to developing the courses are the value of a supportive learning community and the crucial role of various kinds of discourse for students' learning. Both of these highlight the central role of interaction among students and between instructor and students for effective development and integration of learning for the individuals in a course. The interactions include building off one another's ideas—not simply passing on a piece of knowledge from one person to another.

To apply these learning theories to Math 308 and 309, the courses rely heavily on groupwork, using manipulatives, justifying reasoning in writing and in discussions, and developing concepts from ideas generated in class. All of these are based on our theoretical framework, and almost all are a challenge to replicate on-line. We are modeling the methods that we hope our students will use when they teach mathematics to elementary school students. For this reason and because we believe these methods are the most effective for teaching math to our students, we did not embrace the idea of teaching these courses on-line. The live classes have students sitting and working in groups using manipulatives while conducting an activity. Students discuss possible strategies with each other and even show each other how to use the manipulative. The teacher checks in with the groups to see that they understand the activity and are working on it properly, and facilitates a discussion at the end of the activity that highlights the significant mathematical concept generated from the activity.

Description of the On-line Courses

The on-line courses cover the same material and are on the same schedule as the live courses. Each weekly module comprises a description and a list of outcomes for the module, discussion board questions, an individual assignment, a collaborative assignment, and a journal assignment. Each element is described below:

1. Module outcomes and introduction. Module outcomes provide students with the goals for their learning.

2. Discussion Board: Module Openers, Clarifications, and Hint. Each week students begin the work for the week by participating in the discussion page. Introductory questions or activities are posted. To receive full credit students need to do the following: respond to one of the original questions, post their own question (related to a reading, video or the discussion), and post a thoughtful response to someone else's comment or question. The discussion board is also where students post "help" questions about assignments, so that everyone can read the question and responses. Fellow students or the instructor may reply to the questions.

3. Individual Assignments. Each module has a number of readings to complete. The reading both delivers mathematical content and gives context to the activities, and model applications. Associated individual questions are assigned to check for comprehension. Individual activities may include any or all of the following each week: readings, problem sets, interactive on-line exercises, video viewing. They can be started anytime in the week. Often the collaborative activities provide knowledge and tools needed to complete the individual problem sets.

4. Collaborative Activities (Teams of 2 to 3). Each of the collaborative activities begins with a preparatory component that should be done alone to prepare the student for a discussion with their team. The collaborative activities require that students make and test conjectures, and articulate concepts and procedures. Each student works through the collaborative activities on his or her own, then exchanges papers with team members. Through email or telephone, the team discusses its work and makes individual revisions. Each person submits a paper individually with a paragraph at the end explaining how it changed after the team's discussion.

5. Journals. Journals are completed after completing the other activities. They require students to reflect on the week's readings and activities. Journals provide closure to the activities by requiring students to reflect upon the activities and the personal relevance of the subject matter. The ability to think critically and to communicate that thinking is central to the study and practice of all disciplines. These courses involve a considerable amount of both thinking and writing. Through writing students generate, develop, organize, modify, critique, and communicate their ideas. Writing helps students discover what they know, what they don't know, and what they need to know.

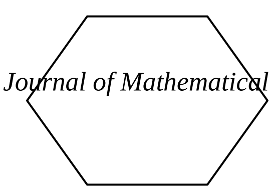
Challenges of an On-line Environment and Attempts to Address Them

Use of Manipulatives

We anticipated that one of our greatest challenges of teaching on-line would be replicating the use of manipulatives in class. It has turned out that the use of manipulatives has been the simplest challenge to overcome. We use a combination of strategies to provide students with a similar experience of developing concepts through the use of models of concrete materials. First, we have students buy key manipulatives (such as geoboards) that cannot easily be constructed by the student. In addition, we have students use everyday household items (such as cups, beans, toothpicks) for some activities. Finally, we use downloadable pictures of manipulatives that can be cut to look like actual manipulatives and we use interactive computer applets for some types of modeling. These "copies" of manipulatives are somewhat controversial. They can serve some of the same purposes as hands-on materials (and in some cases provide even better models), but they have limitations. Seeing a diagram on the screen is different from actually manipulating concrete materials. Also, we like to model in class manipulatives the students may use with their students in elementary and middle school. If our students do not gain experience with them in our classes, they likely will not know how to use them and will not use them with their students.

An example is learning fractions. Concrete manipulatives and computer applets both provide useful models for learning fractions. For example, pattern blocks can be used to divide a shape that measures "1 unit" into any number of fractional pieces. These activities help students understand the meaning of the numerator and denominator (see Figure I). For the on-line class, students can download pictures of colored pattern blocks and cut them out. These can be used in the same way as concrete pattern blocks. Disadvantages of the cutout tools are that they require time to cut them and it is not clear that students are actually using them the way we do when they are done in a group/class setting. The importance of having students use the manipulative as intended is to have them experience the use of manipulatives the way their future students might.

Figure I
Pattern Block Fraction Problem: Given the part and the fraction, find the whole.



2/5 Find the whole.

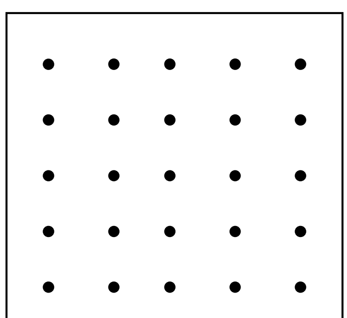
Groupwork

From groupwork we want students to learn to communicate their own ideas and listen to each others', and to experience the power of collaboration for learning to solve complex problems. For the on-line class we thought we could achieve these same results by using the discussion board and by having students share some of their work with a partner each week. Both methods have been productive, but they still have not adequately achieved the benefits of collaborative problem-solving students in our face-to-face classes experience. Having an instantaneous conversation is very different from receiving feedback hours or days later. In addition, the on-line environment does not adequately model the interactions they will engage in as teachers—one of the major rationales for the pedagogies we use.

Journal of Mathematical Sciences & Mathematics Education

An example of a groupwork activity that works in both environments, but differs in its process is "How many different sized squares can you make on a geoboard?" (see Figure II). First, on-line students need to own a geoboard. They can then attempt this problem themselves and post their results. Other students can add to the posted results. A challenge is that students who begin the work later in the week have less incentive to actually figure it out for themselves. They can simply look at the postings. If the problem is done as a collaborative assignment, an individual student might spend too much time stuck after the first few squares. In the live class different students in the group will find different squares. In addition, the teacher can announce periodically the maximum number of squares found so far so that groups know they need to keep looking. The overall time will be limited because students can share within their group, and eventually groups can share with each other. Together all students are more likely to find all of the squares and to find a way of knowing for sure that they have all of them.

Figure II
Geoboard Problem: How many different size squares can you make on a geoboard?

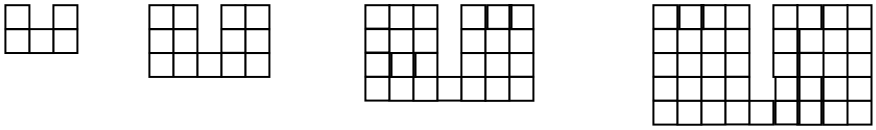


Developing Concepts

A related challenge for the on-line class has been developing concepts from ideas generated in class. In the live classes, we do not show students how to do a problem and then have them practice. We do activities that typically lead them to share their work and discuss key concepts. Instructors can determine during the activity if there are any major misconceptions and lead the discussion to address them, and can also capitalize on students' own ideas for developing the desired concepts. In the on-line environment, we cannot easily lead those types of discussions in a timely manner. Again, the discussion board and assignment sharing with a partner were intended to overcome this challenge, but they have not adequately done so. Some students can be on the totally wrong track all week when working on an assignment. At best, they do not find this out until they receive feedback from their partner the day before the assignment is due. At worst, they do not find out until they have submitted their work and received their grade a week later.

An example is developing algebraic expressions for the number of tiles in a growth pattern of tile sequences (see Figure III). For these problems we provide the on-line class with some examples with explanations of reasoning, but some students still try only to relate a figure to the previous figure in the sequence (rather than to the term of the sequence). That is, they extend the pattern, but do not see the relationships necessary for writing an equation. They develop incorrect expressions for these, but do not understand enough to see that they are incorrect. Those students miss the major concept of the module—looking for a relationship between independent and dependent variables. This also occurs in a live class, but we are better able to address it as it arises and to achieve the learning outcome we are after for that day. We become aware of the students' thinking as we walk around the room observing the groups work and as they share their work with the class. We can therefore challenge students on the spot to think differently, and they learn from their peers' explanations as well. In an attempt to simulate this for the on-line class, we have begun to post "sample student work" after each module so that students can see some of their classmates' work and reasoning. However, it is still not timely enough, nor sufficiently responsive to individual learning needs. To address these issues we have also begun giving the on-line students more completed examples than in the live class, but even these have not solved the problem, and it drifts from our constructivist foundation of the courses.

Figure III
Sample Tile Sequence Problem: Find an expression that represents the number of tiles in the Nth term of this sequence.



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Unexpected Benefits of the On-line Environment

Sense of Community and Peer Supportiveness

The on-line classes have developed a great sense of community. One reason is that in the on-line classes we require students to provide personal profiles so that each student knows something personal about the others in the class, and about their feelings about Math. In a live class, students might share these feelings, but there isn't time for many responses and after that day of sharing, the comments are not revisited. In the on-line environment, profiles are left up all semester. Students wrote very candidly about their feelings about math and many students responded to each others' statements. In addition, shy people are more likely to post these statements than to talk about them in front of a class.

In addition, some students who are shy in class feel much more comfortable participating in a discussion board. They will both pose and answer each others' questions there. Students tended to use a supportive tone in the discussion board. They made statements that sounded like they really wanted each other to understand the math, and they could empathize with their difficulties. This happened much more rarely in the live class.

Peer Feedback

Most students very much appreciated seeing other students' methods for solving problems. They also appreciated receiving feedback from a peer. Unfortunately there were a few students who were so insecure about their work, they resented sharing it. There were also a few students who did not spend much time reviewing their partner's work, and thus, did not provide much helpful feedback. Though this was somewhat eliminated by grading their collaboration.

Comprehensiveness of Write-ups

In both types of classes students are required to always show and explain their work on problems. A scoring rubric is used to assess the mathematical correctness and clarity of explanations. To receive full credit students' work must demonstrate full grasp of the central mathematical ideas and must clearly communicate the student's thinking. Students are often not used to writing in a mathematics class, and thus, have a difficult time writing about their reasoning. In general, the on-line students' write-ups have been more thorough, perhaps because writing is their only medium for communication.

Students in the on-line courses have taken other courses on-line (though not necessarily math courses), and are used to relying on writing for their communication.

Use of Interactive Technology

Because we needed to find alternative ways to model problems in an on-line environment, we searched for and developed interactive tools that help students "see" mathematical concepts. Some of these tools provide better models than the methods used in the live classes (usually drawings). Therefore, we have incorporated some of the interactive tech tools into the live classes too. An example using applets to develop conceptual understanding is with dividing fractions. In the example two-thirds divided by one-seventh, the applet can show a rectangle that has shaded two columns out of three equal sized columns. Then the computer shows splitting another congruent rectangle with one row out of seven shaded. It then puts in both types of gridlines in each rectangle to show same size pieces, and finally counts out moving pieces from the one-seventh to the two-thirds, showing that one-seventh fits into two-thirds, four two-thirds times. The applet can model fitting pieces from one-seventh into two-thirds more dynamically than paper and pencil modeling. We have added the use of the applets and some other technology to our live classes after seeing the benefits in the on-line class.

Journal of Mathematical Sciences & Mathematics Education

Conclusions

While we did not look forward to offering Math 308 and 309 on-line and were concerned that the courses would lose their spirit in this environment, we have found strategies that have partially addressed our concerns and that have provided some unexpected benefits. A key feature of both courses that we have been able to maintain (and sometimes improve) in the on-line classes is using manipulatives and models to develop mathematical concepts. We continue to look for methods that encourage collaboration and a constructivist pedagogy in an on-line environment.

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