The Exclusion of the Students' Dynamic Misconceptions in College Algebra: a Paradigm of Diagnosis and Treatment

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Abstract

We refer to dynamic misconceptions in college algebra as systematic errors, which are the consequences of misapprehensions and misconstructions of algebraic concepts and skills. In summer of 2004 an experimental study was designed and implemented to identify students' misconceptions in college algebra. In addition, a method was devised to eliminate the misconceptions. We employed a selected response assessment instrument to evaluate the students' knowledge in college algebra. Furthermore, we utilized an uncomplicated approach to measure the participants' degree of certainty of their selections. An item analysis of each answer in conjunction with the degree of certainty for that response directed us to determine the student's miscomprehension and/or misinterpretations of the concepts. As a result we identified the participant's level of knowledge (appropriate, improper, or misconception) with reference to the measured information and/or skills. We utilized a combination of instructional strategies such as individual tutoring, peer tutoring, cooperative learning groups, and related computer programs to assist the students to discern their misconceptions. As a final point, we re-taught the misunderstood concepts and skills.

Background

The literature suggests that assessment of students' advancement in mathematics is interwoven with the learning and teaching of this discipline. Stiggins (1997) writes, "Assessment and instruction can be one and the same if and when we want them to be". Glatthorn, et al. (1998) state, "the achievement cycle begins with curriculum and moves from there to assessment and then into instruction". De Lange (1999) expresses, "New views of assessment call for tasks that are embedded in the curriculum, the notion being that assessment should be an integral part of learning process rather than an interruption of it". The objectives of assessment in mathematics are the followings:

- To accumulate data regarding students' learning and achievement in mathematics.
- To analyze this information for feasible revising of the instructional design, delivery, and possible re-teaching of the concepts which students did not attain the expected proficiency in.
- To provide feedback to students, parents or guardians, and school administrators.

The widely used instruments to assess students' knowledge and achievement in a subject matter are standardized test in the form of Selected Response Assessment (multiple choice). These instruments are constructed using a psychometric model. The aim of this model is to be as objective as possible. Nevertheless, there always remains a doubt en route for the possibility of guessing the correct answer or unintentionally choosing the proper response by the student. Taking into consideration the 20% probability of guessing or coincidently selecting the correct response in a multiple choice question with five possible answers, the scores obtaining from such instruments would not endow us with a precise assessment of students' achievement in mathematics. Consequently, we are unable to present an accurate feed back to students, parents or school administrators. Moreover, it would not supply us with adequate information to assist students to triumph over their deficiencies by revisiting the contents. The precise information is essential to analyze and demonstrate student's mastery of required concepts and skills discussed in such instruments. To re-teach a mathematical skills effectively we should investigate if the deficiency is the end result of students' lack of knowledge or it is the consequence of their misapprehensions of the mathematical concepts and skills. In the present document we refer to such misapprehensions and misinterpretations as misconceptions. Since misconceptions in all probability are components of an imperfect cognitive structure shaped because of erroneous knowledge instructed to the students by educators and/or students' life experience, it is indispensable to determine the sources of students' misconceptions. Mestre, J. (1989) writes, "Misconceptions are a problem for two reasons. First they interfere with the learning when students use them to interpret new experiences. Second, students are emotionally and intellectually attached to their misconceptions, because they have actively constructed them. Hence, students give up their misconceptions, which can have such a harmful effect on learning, only with great reluctance". Hasan, et al. (1999) claim, "Misconceptions are strongly held cognitive structures that are different from the accepted understanding in a field and that are presumed to interfere with the acquisition of new knowledge". The treatment to prevail over lack of knowledge requires re-teaching, reconstructing, and reinforcing the concepts and skills which students are deficient in. Meanwhile, the treatment to overcome the misconception requires the elimination of the misconception followed by re-teaching, reconstructing and reinforcing the correct concept and skills.

As mathematics educators it was of our interest to search for a reliable and an effective strategy to assist us to interpret the information obtained from the Selected Response Assessment instruments. Furthermore, we were concerned to not only discover a strategy to eradicate the possibility of guessing the correct response by students, yet assist us in identifying the students' deficiencies as well.

The Method

To achieve this goal we employed a method to measure the students' degree of certainty (DC) regarding to the correctness of their responses. We requested the students select a response for each question, as well as, indicate, on a scale 1-4, how assured they were of their choices. We asked the students to enter a numerical code, 1 through 4, in a space provided beneath each question to indicate their degree of certainty for that particular question. Table I illustrates the characteristics for any given entry code.

Table I
The Characteristic of Responses for Various Entry Degrees of Certainty

Entry Code	Characteristic of the Response
1	They were guessing the response
2	They were not sure about the correctness of the answer
3	They were almost sure with reference to the correctness of the reply
4	They were certain of the correctness of the response

There are 8 possible combinations of different answers collectively with various degrees of certainty for each distinct question in this scheme. Table II demonstrates the possibilities of a student's response in conjunction with the degree of certainty (DC) for a particular question. The table also illustrates the diagnosis of the student's deficiency as well as the appropriate treatment.

Table II
The Possible Combinations of a Student's Response with Various Degrees of Certainty as well as Diagnosis and Treatment for Each Distinct Case

P	Answer	DC	Diagnosis	Suggested Treatment
1	Correct	1	No Knowledge	Re-Teach the Concept
2	Wrong	1	No Knowledge	Re-Teach the Concept
3	Correct	2	No Knowledge	Re-Teach the Concept
4	Wrong	2	No Knowledge	Re-Teach the Concept
5	Correct	3	Proper Knowledge	No Treatment Necessary
6	Wrong	3	Misconception	Remove the Misconception, Re-Teach
7	Correct	4	Proper Knowledge	No Treatment Necessary
8	Wrong	4	Misconception	Remove the Misconception, Re-Teach

P= Possibility DC= Degree of Certainty

Item analysis of a group of students' responses to each particular problem directed us to 4 possible combinations of answers in concert with the mean of various degrees of certainty. We employed the following formula to calculate the average degree of certainty of a group of students for any particular question: $ADC = n^{-1}\left(\sum_{1}^{n} DC\right)$. Where ADC is the group's average degree of certainty for any given response, DC is the degree of certainty of any single student for that particular response, and n is the number of the students forming the group. Table III demonstrates different possibilities of ADC in conjunction with the response for any particular question. The table also indicates the diagnosis and suggested treatments for each possibility.

Table III
The Possible Combinations of a Group's Responses with Average Degrees of Certainty as well as Diagnosis and Suggested Treatment for Each Distinct Case

P	Answer	ADC	Diagnosis	Suggested Treatment
1	Correct	ADC<2.50	No Knowledge	Re-Teach the Concept
2	Wrong	ADC<2.50	No Knowledge	Re-Teach the Concept
3	Correct	ADC>2.51	Proper Knowledge	No Treatment Necessary
4	Wrong	ADC>2.51	Misconception	Remove the Misconception & Re-teach

P=Possibility ADC= Average Group's Degree of Certainty

The Experimental Study

Two classes of College Algebra students participated in our experimental study (54 students). The instrument used in the study was selected from the college algebra textbook publisher's recommended assessment. The instrument was consistent with the content information that was presented to the students. Therefore, the instrument was valid because it exactly measured what it was supposed to measure. The test consisted of Polynomials and Rational Functions, Exponential Functions and Their Applications, and logarithmic Functions and Their Applications. The instrument contained 25 multiple choice questions with maximum possible score of 25 points. We assigned the numerical value of "1" for each correct response and "0" for every wrong answer. The followings are samples of the instrument as well as the direction used in the present study:

Direction: Mark the one alternative that best answerers the question. For each selected response provide a degree of certainty from 1 – 4 such that (*1*) represents ***total guessing***, (*2*) ***not being sure***, (*3*) ***being almost sure***, and (*4*) ***being certain*** of your response.

- **Solve for *x*:** $(x-0.75)^{2.5}=32$

  (a) $32^{0.4}+0.75$

  (b) $32^{0.4}-0.75$

  (c) $32^{2.5}-0.75$

  (d) $32^{2.5}+0.75$

  (e) $32^{2.5}+1.25$

  Degree of Certainty:

1	2	3	4

- **We increase the length of a cubic container by 3 and decrease its height by 1. Calculate the width of the container if its volume is 6 cm^3.**

  (a) $\sqrt{3}$
  (b) 9
  (c) 2
  (d) $-\sqrt{3}$
  (e) 1

  Degree of Certainty:

1	2	3	4

- **We deposit $15000 in a saving account. The monthly compounding interest is 2.1%. Calculate the amount of the interest we will earn after 9 years.**

  (a) $ 18120.61
  (b) $ 3161.21
  (c) $ 3120.61
  (d) $ 1866.16
  (e) $ 4077.23

  Degree of Certainty:

1	2	3	4

- **It takes 100,000 years for 20 grams of a radioactive element to be reduced to 18 grams. Calculate the half-life of this element.**
  (a) 526820 years
  (b) 326941 years
  (c) 1053651 years
  (d) 657881 years
  (e) 693147 years

  Degree of Certainty: | 1 | 2 | 3 | 4 |

We assessed the students' responses; in addition, we evaluated their answers in conjunction with their degrees of certainty. Table IV displays the item analysis for one individual student. Table V exhibits the item analysis for a group of students.

Table IV
A Sample of Item Analyses of a Student's Responses Combine with Degree of Certainty as well as Diagnosis and Treatment

Q	G	DC	Diagnosis	Suggested Treatment
1	1	4	Proper Knowledge	None
2	1	4	Proper Knowledge	None
3	1	4	Proper Knowledge	None
4	1	4	Proper Knowledge	None
5	1	4	Proper Knowledge	None
6	0	4	Misconception	Remove the Misconception, Re-Teach
7	1	3	Proper Knowledge	None
8	1	3	Proper Knowledge	None
9	1	3	Proper Knowledge	None
10	1	3	Proper Knowledge	None
11	1	3	Proper Knowledge	None
12	1	4	Proper Knowledge	None
13	1	4	Proper Knowledge	None
14	1	3	Proper Knowledge	None
15	1	3	Proper Knowledge	None
16	1	2	No Knowledge	Re-Teach the concept
17	0	1	No Knowledge	Re-Teach the concept
18	0	4	Misconception	Remove the Misconception, Re-Teach
19	0	4	Misconception	Remove the Misconception, Re-Teach
20	1	3	Proper Knowledge	None
21	0	1	No Knowledge	Re-Teach the concept
22	0	3	Misconception	Remove the Misconception, Re-Teach
23	0	3	Misconception	Remove the Misconception, Re-Teach
24	0	4	Misconception	Remove the Misconception, Re-Teach
24	1	2	No Knowledge	Re-Teach the concept
24	0	1	No Knowledge	Re-Teach the concept

Q = question G = Grade DC = Degree of Certainty

Table V
The Item Analyses of a Group's Responses Combine with Average Degree of Certainty as well as Diagnosis and Treatment

Q	AG	ADC	Diagnosis	Suggested Treatment
1	1.00	3.82	Proper Knowledge	None
2	1.00	4.00	Proper Knowledge	None
3	1.00	4.00	Proper Knowledge	None
4	0.96	3.00	Proper Knowledge	None
5	0.88	3.62	Proper Knowledge	None
6	0.16	3.20	Misconception	Remove the Misconception & Re-Teach
7	0.78	3.41	Proper Knowledge	None
8	0.81	3.50	Proper Knowledge	None
9	0.82	3.71	Proper Knowledge	None
10	0.76	3.64	Proper Knowledge	None
11	0.96	3.82	Proper Knowledge	None
12	0.68	3.42	Proper Knowledge	None
13	0.82	2.82	Proper Knowledge	None
14	0.98	3.12	Proper Knowledge	None
15	0.78	2.88	Proper Knowledge	None
16	0.10	2.02	No Knowledge	Re-Teach the concept
17	0.42	1.12	No Knowledge	Re-Teach the concept
18	0.56	3.80	Misconception	Remove the Misconception & Re-Teach
19	0.43	3.64	Misconception	Remove the Misconception & Re-Teach
20	0.96	3.50	Proper Knowledge	None
21	0.23	1.16	No Knowledge	Re-Teach the concept
22	0.38	3.20	Misconception	Remove the Misconception & Re-Teach
23	0.42	3.74	Misconception	Remove the Misconception & Re-Teach
24	0.42	2.12	No Knowledge	Re-Teach the concept
24	0.10	1.36	No Knowledge	Re-Teach the concept

Q=Question AG=Average Grade ADC=Average Degree of Certainty

The data, suggested that the majority of our students had misconceptions regarding the mastery of the subject matters discussed in items # 6, 18, 19, 22, and 23. In the present investigation the misconceptions were gave an account for the items involving the Exponential and Logarithmic Functions and Their Applications. The item analysis indicated that many of the subjects misunderstood the rules of logarithms and failed to employ the correct rules in problem solving. Our task was to remove these misapprehensions and misinterpretations. As mathematics educators and classroom teachers we have experienced the students' resistance to discern their misconceptions. Furthermore, re-teaching the concepts and skills requires us to differentiate between the students' misconception and their improper knowledge. We modified the design of our instructional content and delivery. Moreover, we employed such learning strategies as: small cooperative group, peer tutoring, individual tutoring, and utilized related instructional software to further encourage the students to allow their misconceptions to surface. Finally we presented the students with the correct concepts and skills.

A remarkable characteristic of the present study was its simplicity and straightforwardness. This attribute encouraged the students to unveil their misconceptions as a part of their assessment. Hence, they assisted us to discover and extract the misconceptions. We strongly recommend that mathematics educators of grades 9 – 16 utilize this strategy to examine the students' genuine mathematical knowledge and to identify their misconceptions.

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