Correlation between Students' level of Understanding Geometry According to the van Hieles' Model and Students' Achievement in Plane Geometry

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Abstract

Pierre van Hiele and Dina van Hiele-Geldof developed a model of learning geometry in late 1950's. The model is applied by identifying the students' level of thinking, designing the instruction for their particular stage, and assists them to advance to the next level.

An experimental study was designed and implemented by the author to explore the existence of a relationship between the van Hieles' level of understanding geometry and achievement in plane geometry. One hundred-sixty nine students participated in the experiment. A pretest was administered to all participants at the beginning of the semester. The pretest consisted of the following two selected response assessment instruments: The "Plane Geometry National Achievement Test" and the "Van Hieles' Geometry Test". The same battery of tests was employed as a posttest and was administered to the participants after 6 weeks of instruction.

Measuring the linear relationship Between Students' level of Understanding Geometry According to the van Hieles' Model and Students' Achievement in Geometry we found a correlation coefficient of 0.8665 for the posttest. The results indicated that there was a strong positive correlation between the advancement of the van Hieles' level of understanding geometry and achievement in geometry.

The hierarchical nature of the van Hieles' Model has significant implications for teaching geometry. We suggest that educators responsible for geometry instruction and professionals in charge of teacher training programs incorporate the principles upon which the van Hieles' model is based into instructional and curricular design.

Background

Pierre van Hiele and Dina van Hiele-Geldof developed a model of learning geometry in late 1950's. Theirs is one of the most influential research works for the teaching and learning of school geometry. Shaughnessy and Burger (1985) state, "From classroom observations, the van Hieles felt that the students passed through several levels of reasoning about geometric concepts." In addition, the van Hieles' model for levels of understanding geometry make a significant advance beyond the use of basic Gagné learning hierarchies because of their identification of qualitatively distinct levels and the role of reification in progressing from one level to the next. Moreover, it appears that the van Hieles' findings are direct implications of the theories of cognitive learning in geometry education or at least pursue the same principles.

Van Hieles' model consists of five distinct levels: Visualization, Analysis, Informal Deduction (Order), Deduction, and Rigor. Based on van Hieles' scheme, instruction is designed according to a scheduled sequence. The objective is to lead the students to a higher level of thinking. A brief description of the levels of understanding geometry based on the van Hieles are as follow:

- Level 0: Visualization, students see geometric figures as a whole, but do not identify the properties of figures as at the next level.
- Level 1: Analysis, student can identify the figures, their features and characteristics properties even though they do not understand the interrelationship between different types of figures, and they also cannot fully understand or appreciate the uses of definitions at this level in contrast to understanding and performance at the next level.
- Level 2: Informal Deduction (Order), students can understand and use definitions. Concept nesting is understood and accepted as in the case of every square being a rectangle. They are able to make simple deductions and may be able to follow formal proofs but do not understand the significance of working in an axiomatic system and are not able to construct proofs meaningfully on their own at this level.
- Level 3: Deduction, students can construct proofs at this level as a way of developing geometry theory. The interrelationship between undefined terms, definitions, axioms/postulates, theorems, and proof is understood and used. However, at this level they are limited and not at a level of being able to work in a variety of axiomatic systems, and the rigor of logical and geometrical methods as at the next level.
- Level 4: Rigor, students understand logical and geometrical methods. They are able to work in a variety of different axiomatic systems. They are able to appreciate the historical discovery of non-Euclidean geometries and the freeing of geometry from physical materialism. They understand how a multitude of distinctly different geometries can exist, be developed, investigated, and used. They can also appreciate how a particular geometry (e.g., Euclidean) can be studied from different perspectives with different methods (synthetic, analytic, transformationally).

The van Hieles' model is utilized to identify the level of students' thinking by means of engaging them in conversations about geometric topics, then designing the instruction for their particular level and helping them to advance to the next level. Based on the van Hieles' model, instruction must be designed and delivered consistence with a sequence of phases. Their succession of teaching-learning activities will guide the learners to a higher level of thinking. The following is a simple description of the phases according to the van Hieles' design.

- Inquiry, discussion between teacher and student concerning a geometric topic.

- Direct Orientation, exploring the properties of figures by experimentation.
- Explication, forming a network of relations regarding the geometric topics.
- Free Orientation, challenging the students independently.
- Integration, incorporate students' knowledge about specific topic.

After Phase 5 students advance to the next higher level. The teacher repeats the procedure again at the new level. Since much of learning originates from inside the learner, students' higher level of thinking leads them to a concept formation strategy, which is a mean of pulling discrete items together into larger conceptual schemes. This strategy calls for learners to examine their information and organize it into concepts and to manipulate those concepts.

Methods and Procedures

One hundred-sixty nine, 15-17 year old students attending a predominantly African American high school participated in an experiment conducted by the author. A pretest was administered to all participants. The pretest consisted of the following selected response assessment instruments: The "Plane Geometry National Achievement Test" and the "Van Hieles' Geometry Test". After 6 weeks of instruction a posttest was administered to the participants. The scores obtained from each instrument were analyzed.

To examine the existence of a linear relationship between the levels of understanding geometry and achievement in geometry a triangulation strategy was used to collect data and assess the subjects. Results from the following two data sources were combined in the research design:

- "Van Hiele Geometry Test", VHGT, which measured the participants' level of understanding of geometry.
- "Plane Geometry: National Achievement Test", PGAT, which measured the participants' level of achievement in geometry.

To determine the existence and strength of the relationship, the correlation coefficients were calculated for both the pretest and the posttest. The correlation coefficients are shown in Table I.

Table I
Correlation Coefficients between VHGT Scores and PGAT Scores

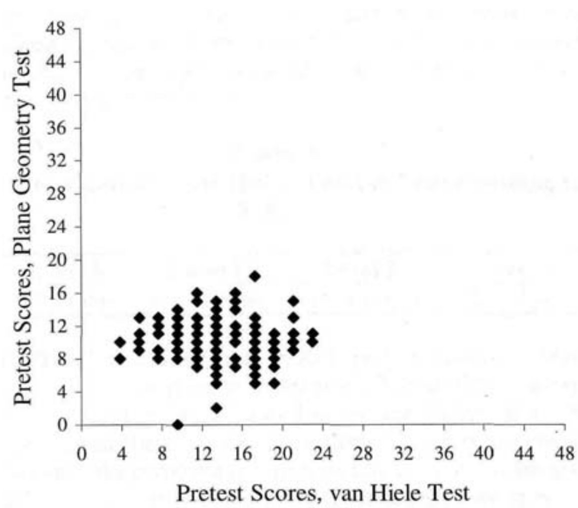
	PGAT Pretest	PGAT Posttest
VHGT Pretest	r = 0.0288	
VHGT Posttest		r = 0.8665

Measuring the linear relationship, the calculated correlation coefficient of 0.0288 was determined for the scores obtained from the VHGT pretest and the

PGAT pretest. The data collected from the pretest indicated a mean score of 7.301 for VHGT and a mean score of 10.25 for PGAT. The mean scores of the pretests placed the participants at van Hieles' level one, Analysis. Given that level one is characterized by the identification of figures and their properties without understanding the definitions or interrelationships between different figures, the achievement level measured by the PGAT would be outside the domain of such a basic level of understanding. Therefore, no correlation could be derived from the scores obtained from the pretest.

Considering the multiple-choice format of the instruments, it is conceivable that the scores obtained occurred by chance. Using the VHGT, participants were given five choices for each of the twenty-five questions. Participants would have had a 20% chance of selecting the correct answer by guessing. This would generate a score of five, which represents the borderline score for the van Hieles' level one. Therefore, no information can be derived at such a low level. Using the PGAT, participants were given five choices for each of the forty-eight questions. Students would have had a 20% chance of selection the correct answer by guessing. This would generate a score of 9.6, which is on the borderline of the mean of the PGAT pretest. Therefore, no information can be derived from this score. The graphical representation of the relationship between the VHGT pretest scores and PGAT pretest scores is provided in Figure I.

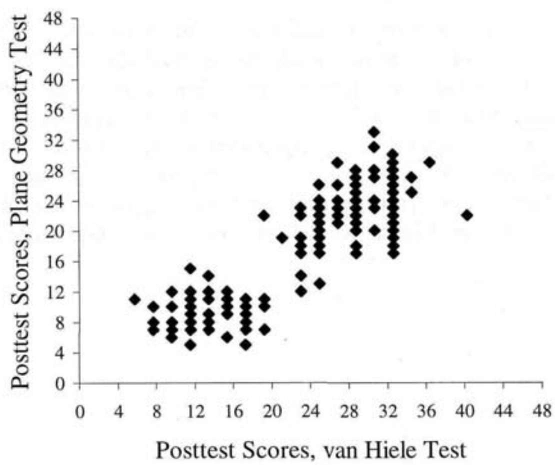
Figure I
Relationship between VHGT pretest scores and PGAT, (r = 0.0288)



A correlation coefficient of 0.8665 was found for the scores obtained from the VHGT posttest and the PGAT posttest. A linear relationship between the scores of the VHGT posttest and the PGAT posttest was established. Therefore, we concluded that there exists a strong positive correlation between the scores of

VGHT posttest and the scores of PGAT posttest. The graphical representation of the relationship between the VHGT posttest and PGAT posttest is shown in Figure II.

Figure II
Relationship between VHGT posttest scores and PGAT, (r = 0.8665)



Results and Conclusions

The strong correlation between VHGT posttest and PGAT posttest (r = 0.8665) indicated that there was a relationship between achievement in geometry and advancement in the van Hieles' level of understanding geometry. The rubric used for classifying the van Hieles' level of understanding geometry is represented in Table II

Table II
The Rubric for the Classification of van Hieles' Level of Understanding Geometry for the VHGT

Van Hieles' Level	Level 0	Level 1	Level 2	Level 3	Level 4
Scores	0-5 Points	6-10 Points	11-15 Points	16-20 Points	21-25 Points

Applying the VHGT rubric on the VHGT pretest indicated that the participants were functioning at van Hieles' level one prior to instruction. The statistical analyses of the VHGT posttest indicated that these participants advanced to the van Hieles' level three (Deduction) following the treatment. According to the van Hieles' scale, the students advanced two levels. The mean difference between the participants' progress through the van Hieles' levels for pretest and posttest was 6.0327 points in favor of the posttest. It is important to consider that although each van Hiele level has a range of five points, the difference between scores at the upper limit of one level and scores at the lower limit of the higher conceptual level is only one point. In this context, although the mean difference of pretest and posttest was not ten points, nevertheless, the two test results placed them into different levels.

Implications for Practice

In exploring the existence of a relationship between the van Hieles' level of understanding geometry and achievement in plane geometry, we found that there existed a correlation between the van Hieles' level of understanding geometry and student achievement in plane geometry. The results indicated that there was a direct correlation between the van Hieles' level of understanding geometry and achievement in geometry. We recommend that school geometry teachers revise their instructional methods to utilize the van Hieles' strategies in planning and delivering lessons. Textbooks, technology, and other instructional materials and equipment are needed that support the van Hiele model. Curriculum standards need to be revised in light of these and similar research findings. In addition, we suggest professionals responsible for teacher education programs incorporate the principles upon which the van Hieles' model is based into courses dealing with instructional methods and curricular design.

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