The Mathematical Preparation of Secondary Teachers: A Call for Research

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Abstract

When the mathematical preparation of secondary teachers is deficient, it has been shown to impact both their instructional practice and their students' achievement. Teacher deficiencies are not conclusively related to the quantity of mathematics courses that they have taken, but instead, to the nature of the courses and depth of their understandings. This article summarizes applicable research supporting this argument and national recommendations for meeting the mathematical needs of prospective secondary teachers. Various curricular efforts currently responding to the research and recommendations are surveyed and critiqued, culminating in a call for mathematicians and mathematics educators to engage in much needed, rigorous research.

Introduction

The primary responsibility of mathematically preparing secondary teachers has always fallen to mathematics departments, which have sometimes embraced these students but at other times treated them as less worthy step children. Additionally, there is evidence of neglect among mathematics education research. When reviewing literature on the mathematical preparation of teachers, it is challenging to find rigorous research specifically targeting secondary teachers. Although some may argue that researchers involved with reforming secondary mathematics teaching can make inferences from research at elementary and middle grade levels, more attention should be given to the special nature of the mathematics needed to teach high school. Therefore, this paper seeks to provide the extant literature and standards that frame the mathematical preparation of secondary teachers and, within that framework, analyze current and ongoing institutional reforms. Together, these may serve to inform departmental discussions regarding the preparation of undergraduates for secondary teaching careers.

Framing Mathematics Needed for Teaching

Mathematics education researchers have been working with existing research to develop a framework that accurately articulates mathematical knowledge needed for teaching. This framework is critical to assessing teachers' deficiencies, which have been shown to impact the way they teach and ultimately their students' achievement. The mathematics community has also produced detailed recommendations for teacher preparation programs. What follows summarizes and analyzes the similarities of the efforts emanating from both the mathematics and mathematics education communities.

Related Research

Over the decades, mathematics educators have refined their understanding of the type of knowledge teachers require to be successful in the classroom. Ball (1991) posited that teachers must acquire both knowledge *of* mathematics and knowledge *about* mathematics. Knowledge *of* mathematics referred to both the procedural aspects of and underlying concepts. Knowledge *about* mathematics constituted understanding its nature, how mathematical knowledge is created, and what it means to do mathematics. Shulman (1996) argued further that teachers must not only know the subject matter and its structure, but also acquire *pedagogical content knowledge*. In mathematics, *pedagogical content knowledge* would include understanding how students learn particular concepts or strategies, what makes particular notions challenging or simple, when misconceptions which may arise, and which representations should be used in problem solving or discussions.

In elementary mathematics education, Ball, Bass and Hill (2004) extended this theoretical framework when analyzing videos of elementary classrooms mathematics lessons to identified explicit mathematical activities in which teachers engaged. These activities included:

- Articulating explanations that are mathematically accurate and useable for students;
- Creating and evaluating mathematical definitions;
- Using multiple representations for problem solving and mapping between representations;
- Interpreting and evaluating students' mathematical work;
- Responding productively to questions and mathematical curiosity;
- Appraising the quality of instructional materials and making necessary modifications; and
- Posing rich mathematical problems or questions.

(Ball, Bass and Hill, 2004)

These activities illustrate the interwoven nature of the mathematical demands of teaching, making it difficult to distinguish what constitutes mathematical activity and educational activities. This work led to the framework which is now termed *Mathematical Knowledge for Teaching* and is currently being refined throughout ongoing research at the Center for Proficiency in Teaching Mathematics (A joint effort of the University of Michigan and University of Georgia funded by the National Science Foundation). Unfortunately, comparable research has not been conducted at the secondary level; however, Usiskin (2001) has suggested a framework for secondary mathematics based on areas of analysis in which teachers engage during lessons. He argued that teachers must be able to:

1. Analyze concepts: recognize ways of explaining and representing ideas, select alternative definitions and their consequences, understand how mathematical ideas arose and evolved over time, and apply mathematics to a range of fields;

2. Analyze problems: understand alternative approaches to problem solving and have the ability to extend or generalize proofs and problems;
3. Analyze connections: explain links between school mathematics, the broader field of mathematical study and the world.

These two frameworks capture a specialized nature of mathematics needed for teaching, but also note that much of it requires deep knowledge of mathematics and an understanding of how knowledge is created in the field of mathematics. In essence, it calls for a cultural understanding of the nature of mathematics, viewing mathematics as a field of inquiry.

The knowledge required to teach mathematics is, therefore, much deeper than a cursory knowledge of the subject. Unfortunately, teachers' deficiencies in mathematical knowledge have been well-documented but are not necessarily linked to the quantity of mathematics courses taken, a measurable teacher characteristic that has not been conclusively correlated with student achievement (Brown and Borko, 1992; Mewborn, 2003). Instead, mathematical deficiencies are more likely related to the quality and depth of teachers' exposure to mathematics. As Mewborn (2003) notes in a review of literature on teaching and teacher knowledge, teachers have been found to have "a strong command of the procedural knowledge of mathematics, but they lack a conceptual understanding of the ideas that underpin the procedures" (p. 47).

Research has demonstrated that a teacher's conceptualization and knowledge of mathematics has a direct impact on their instructional practice (Goldsmith and Shifter, 1997; Koency and Swanson, 2000; Mewborn, 2003; Stipek, Givvin, Salmon, and MacGyvers, 2001). If teachers learn and view mathematics only procedurally, they may subscribe to the notion that learning mathematics involves only developing procedural proficiency (Goldsmith and Shifter, 1997). For example, teachers who understand the procedural aspects of mathematics but not the underlying concepts and reasoning are more likely to emphasize algorithms and facts in their instruction rather than problem solving and inquiry (Thompson, 1984). Furthermore, teachers' instructional practices and beliefs are strongly influenced by how they were taught and how they perceived mathematics while they were learners (Brown and Borko, 1992; Thompson, 1984).

National Recommendations for Mathematical Preparation

This broader view of the mathematical knowledge needed for teaching, which has thus far been presented in the work of mathematics education researchers, is also consistent with recommendations of the mathematical community. Two documents contain recommendations of national mathematics organizations: (1) *Mathematical Education of Teachers* (*MET*), produced by the Conference Board of Mathematical Sciences (CBMS) with the support of the American Mathematics Society
(AMS) and the Mathematical Association of America (MAA) (CBMS, 2001); and (2) *Undergraduate Programs and Courses in the Mathematical Sciences:*

CUPM Curriculum Guide 2004, created by the MAA Committee on the Undergraduate Program in Mathematics (CUPM, 2004).

<u>*Mathematical Education of Teachers.*</u> A major theme of the *MET* report was to recognize that the mathematical content knowledge needed to teach preschool to 12th grade mathematics was substantive yet different from that needed for other mathematically-related careers. The report challenged the notion that standard mathematics curriculum adequately prepared teachers and recommended that programs be redesigned to assist prospective teachers in making connections between mathematics studied at the collegiate level and their future classrooms. Their recommendations included core mathematics coursework adaptations for secondary teacher preparation, which they indicated "may be as appropriate for all mathematics majors as they are for prospective teachers" (CBMS, 2001, p.143). For example, assignments in an abstract algebra course might be adapted by including a thorough analysis of number systems that extends natural numbers to complex numbers and examines the associated impact on properties that govern them.

The *MET* report also recommended creating capstone courses designed for teachers that would address specific topics in secondary mathematics but at a more advanced level to solidify connections made between collegiate and secondary mathematics. In addition, the *MET* report emphasized the inclusion of historical and cultural perspectives, utilization of various modes of learning mathematics, and development of mathematical thinking or habits of mind. The latter should encourage "(i) asking and exploring interesting mathematical questions; (ii) framing mathematical concepts and relationships in clear language and notation; (iii) constructing and analyzing proofs; (iv) applying mathematical principles in other disciplines"(CBMS, 2001, p. 141). This emphasizes activities related to understanding how knowledge is created in the field of mathematics, similar to suggestions emanating from research in mathematics education.

<u>*CUPM Curriculum Guide.*</u> While the *MET* report focused only upon mathematics teacher preparation, the *CUPM* document included recommendations for all mathematics curricula. Many of its recommendations for teacher preparation echoed the *MET* document, including that all secondary prospective teachers not only complete the course requirements for mathematics majors but also:

> Learn to make appropriate connections between the advanced mathematics they are learning and the secondary mathematics they will be teaching. They should be helped to reach this understanding in courses throughout the curriculum and through a senior-level experience that makes these connections explicit.
> (*CUPM*, 2004, p. 52)

The *CUPM* report endorsed the *MET* report recommendations arguing that the adjustments to content courses needed for education majors would benefit most majors. Interestingly, one of *CUPM* broad recommendations for all mathematics programs addresses the mathematical thinking of all students of mathematics:

> Every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind. More specifically, these activities should be designed to advance and measure students' progress in learning to:
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> - State problems carefully, modify problems when necessary to make them tractable, articulate assumptions, appreciate the value of precise definition, reason logically to conclusions, and interpret results intelligently;
> - Approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures;
> - Read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking.
>
> (*CUPM*, 2004, p 1)

These excerpts illustrate the consistency of the *CUPM* guide with the mathematics needed for teaching suggested by Ball, et al. (2004) and Usiskin (2001) and move beyond specifying particular skills and theorems, emphasizing the need for programs to produce students who can participate in mathematical inquiry. Overall the commonalities of *MET, CUPM,* and mathematics education frameworks suggest that the mathematical preparation of secondary teachers should consist of:

- acquiring core mathematical concepts and skills,
- conceptualizing mathematics as a field of inquiry,
- developing specialized mathematical knowledge for teaching; and,
- connecting collegiate and secondary mathematics.

Responding with Innovations in Mathematical Preparation

Teacher preparation programs have responded to these recommendations by employing innovations which include the development of capstone courses, specialized content courses, companion coursework and modified instructional approaches for traditional coursework. It should be noted that the body of research addressing secondary mathematics teacher preparation is emaciated and there is a significant need to rigorously examine the effectiveness of the following approaches. Therefore, each is offered as a potential avenue to address secondary teachers' mathematical preparation and is presented with questions to promote discussions of the theoretical and practical considerations of implementing similar programmatic changes.

Specialized Mathematics Coursework

In response to the need for specialized mathematics knowledge, some have developed coursework designed specifically for secondary mathematics majors. Usiskin, Marchisotto, Peressini, and Stanley (2003), for example, created two-course sequence that examines secondary mathematics from an advanced perspective and makes explicit connections between collegiate and secondary mathematics. This approach has the potential to address specialized content knowledge, but may also invite questions:

- By substituting traditional coursework with specialized content, are we compromising the acquisition of mathematical understandings and skills that we should expect of prospective teachers as well as mathematics majors?
- Does separation of prospective teachers from traditional coursework inhibit the development of understanding of mathematics as a field of study? If so, does traditional coursework sufficiently address this need?
- Is it practical to offer separate courses for teachers at institutions without adequate resources or enrollment to justify additional coursework?

The first question may be addressed, in part, by viewing mathematics courses for secondary teachers as comparable to courses specialized for engineering, business, or perhaps liberal arts. If courses explicitly address connections between mathematics and a particular discipline, then they should do so without diminishing the level or rigor of the content presented. Addressing the second question is more challenging. If traditional coursework presumably provides prospective teachers with a glimpse of mathematical inquiry, its topics and processes, can an equivalent breadth of knowledge and perspective be delivered with substantively different mathematical content? To answer this requires additional research into how programs cultivate an understanding of mathematics as a field of inquiry, whether the process is intentional or perhaps a by-product of exposure to research mathematicians. The final question entails practical considerations which are incredibly salient since mathematics teachers are often produced by institutions that do not have the capacity to offer separate coursework. This is particularly true in Georgia, where the six public university that offered baccalaureate degrees leading to secondary mathematics teacher certification conferred an averaged 8.1 degrees in 2004 (including one flagship institution that conferred 21 degrees). While offering separate coursework may be optimal for the prospective teachers, it may be unfeasible for many programs.

Capstone Coursework

Both the *MET* report and *CUPM Guide* recommended creating capstone courses for prospective secondary teachers that explicitly connect collegiate and secondary mathematics. While research on the impact of these courses is limited, the approaches and specialized content addressed in them have been perceived by instructors and students as having a positive effect on connecting topics while modeling researched based instructional practices (Shoaf, 2000). Since they are intended as culminating courses and typically offered during the last semester of coursework, capstone courses act as a supplement to existing mathematics program requirements. Because of their supplemental nature, these questions should be considered:

- Can capstone courses alone promote prospective teachers' understanding of connections between collegiate and secondary mathematics?
- Do culminating courses miss opportunities to make connections during core coursework, thereby limiting perceived relevance of traditional content?

Both questions are related to a deeper concern about the motivation of prospective teachers to learn mathematics that they cannot directly connect with their future teaching. The perceived value of mathematical tasks has been shown to be a critical motivational component of acquiring mathematical knowledge, impacting both level of effort and achievement (e.g. if students perceive tasks are valuable, they are more likely to work harder on them) (Pokay, 1996). Creating capstone courses alone may be insufficient since their placement at the end of coursework may be too late to affect the acquisition of mathematics in core classes. These courses need to be part of a comprehensive strategy designed to make connections throughout programs to optimize prospective teachers' motivation to acquire core knowledge.

Companion Coursework

A more recent idea is to create shadow courses that coincide with traditional mathematics coursework. For example, Kehle, Maki, and Nowlin (2005) are developing one credit hour seminars that are taken in tandem with mathematics courses. What they term as "linking courses" are taught in conjunction with calculus, linear algebra and abstract algebra. Designed in a partnership with practicing secondary teachers, the courses make direct links between secondary and collegiate mathematics. This particular strategy appears promising, but research is needed to demonstrate the impact of companion coursework, examining questions such as:

- Do companion courses impact prospective teachers' perceived value of traditional mathematics courses?
- How are prospective teachers' dispositions to engage in mathematics impacted?

Modified Instructional Approaches

The final avenue taken by teacher preparation programs has been to modify traditional coursework by focusing on changing or reforming instructional strategies. For example, Blanton (2001) modified her teaching to encourage discourse in a geometry course by engaging students in discussions and critiques of proofs, then stimulating reflection on their observed discourse patterns. Benson and Findell (2002), in another instance, used a modified discovery approach in abstract algebra that employed cooperative group work to explore group theory. These teaching strategies challenged students to move beyond acquiring skills and concepts to developing notions of what it means to participate in mathematical inquiry. However, the approach of modifying instruction may also be limited:

- Does a focus on teaching strategies make needed connections between secondary and collegiate mathematics?
- Does modifying instruction address the specialized content needed?
- Is it realistic to effectively reform the instructional practices throughout undergraduate mathematics coursework?

Although the explicit connections between collegiate and secondary mathematical *topics* may not necessarily be addressed, modifying instruction does connect *strategies* recommended for teaching mathematics. Additionally, modified approaches may foster viewing mathematics as a field of inquiry, which addresses many aspects of the specialized content knowledge previously delineated. Potential effects, however, must be studied. In response to the final question, a broad effort to reform teaching of undergraduate mathematics and science courses is underway in Oregon, which may provide valuable insights. In that state, Wainwright, Flick, Morrell, and Schepige (2004) are examining the impact of a professional development program on observed classroom practices of undergraduate courses. Though their results are preliminary, some changes in their teaching practices have been observed.

Balancing Mathematical Needs

Each innovative approach has been considered independent of the others, without recognition that each may be implemented within more comprehensive strategies to meet the mathematical needs of prospective teachers. Indeed, a comprehensive strategy is necessary if we hope to offer not only separate courses designed to make explicit connections to secondary mathematics, but also modify existing curriculum and instruction to positively impact prospective teachers' conceptualization of mathematics. While future teachers require specialized content and must make connections between collegiate and secondary mathematics, they also need to understand core mathematics deeply and to recognize the process of mathematical inquiry. These two aspects are equally essential for *all* mathematics majors. Recognizing these commonalities can assist in developing programs that balance the specialized needs of prospective teachers with the needs of mathematics majors who often share much of their coursework.

The survey of innovative approaches clearly reveals that additional research is needed to document their impact. While the approaches are currently perceived as effective, the body of evidence needed to guide programs remains tenuous, at best. As programs consider implementing innovations, it is imperative to conduct accompanying research into the direct impact on prospective teachers. Additionally, the framework of the mathematical needs of teachers has also relied too much on our knowledge of teacher practices in elementary and middle grades. It is critical that parallel research be conducted at the secondary level to validate, critique, and refine our understanding of the mathematical needs of secondary teachers.

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