# The Limitations of Direct Sentence Translation in Algebraic Modeling of Word Problems

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**Abstract**

Solving word problems is probably the eventual objective in mathematics classes. Direct sentence translation strategy is recommended as a major process of solving world problems in algebra classes. This strategy is sufficient for translating a broad range of elementary algebra problems. However, for a number of problems that require multiple steps of establishing relationships among values rather than quantities, this system generates a translation error pattern known as Reversal Error.

In this document we examine phenomenon of Reversal Error in an algebra environment for pre-service teachers. Since the algebraic notations are not self-explanatory in various situations; therefore, the occurrence of reversal error is a product of associating algebraic notations with incorrect number values. The findings of the present study are consistent with the findings reported by: Clement, Lochhead, & Monk (1981) for calculus-level students, Fisher (1988) for introductory-level physics students, and Cohen & Kanim (2005) for calculus-based physics students.



## Introduction

One of the principal purposes of education is to prepare students as problem solvers. As mathematics educators, our goal is to school the future generation of mathematics problem solvers. Slavin (1997) states, “Students cannot be said to have learned anything useful unless they have the ability to use information and skills to solve problems”. In addition, National Science Education Standard, NSES, (1996) points out, “The business community needs entry-Level workers with the ability to learn, reason, think creatively, make decisions, and solve Problems”.

Solving real world problem is a focal challenge for majority of students in mathematics classes. Polya (2004) notes “Solving a problem means finding a way out of difficulty, a way around an obstacle, attaining an aim, which was not immediately attainable. Furthermore, the National Council of Teachers of mathematics, NCTM (2000), states, “Problem solving means engaging in a task for which the solution method is not known in advanced. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort on their thinking”.

Abstract thinking and problem solving is the effect of Formal Operational stage of Piaget's Cognitive Development (1-Sensory Motor, 2-Pre-operational, 3-Concrete Operational, and 4-Format Operational, and Formal Operational). In addition, it is the end results of Robert Gagne's Hierarchical Learning (1-Signal Learning, 2-Stimulus Response Learning, 3-Chaining, 4-Verbal Association, 5-Multiple Discrimination, 6-Concept Learning, 7-Principle learning, and 8-Problem Solving). Moreover, it is the desired outcome of Van Heiles', Levels of Understanding Geometry (1-Level 0, Visualization, 2-Level 1, Analysis, 3-Level 2, Informal Deduction, 4-Level 3, Deduction, 5- Level 4, Rigor). Level-4 of van Heiles' scheme, rigor, is the generalization of geometric concepts and problem solving.



The students not only should learn the mathematical concepts and master the skills of doing mathematics; they should also reason and communicate mathematically and solve real life problems. Problem solving is the first standard to be exploited for students' achievement in mathematical literacy in K-4, 5-8, and 9-12 in school curriculum.

It seems that the most difficult undertaking for majority of mathematic students is to utilize the learned mathematical concepts and mastered mathematical skills in solving the real situation word problems. Slavin (1997) writes "The difficulty of most applications problems in mathematics lies not in the computation, but in knowing how to set the problem up so that it can be solved. Problem Solving is a skill that can be taught and learned".

## Methods and procedures

There are several strategies and methods suggested for problems solving in mathematics. Polya (2004) recommends the following procedures as a how to list:

1. Understanding the Problem (What are you trying to do? What is unknown? What are the data?)
2. Devising a Plan (Draw a figure, table, or graph for the problem. Can you relate to the problem? Find the Connection between the Data and the Unknown. Obtain a plan of the solution.)
3. Carrying out the Plan (Check each step and make sure that the step is correct.)
4. Looking Back (Examine the solution and check the answer.) (How to Solve It, Polya 2004, p. ix )

Operational fragments in "Devising a Plan" of Polya's list is our main concern in Algebra for P-8 Pre-service Teacher course. According to majority of textbooks' recommendations and mathematics teachers, Devising a Plan to solve algebra real life and word problems requires that students have the ability to translate linguistic statements into algebraic equations and further, manipulate the algebraic equations and solve them. Such translation is as translating one natural language to another. Students generally use key English words or phrases that are associated with their most common used in mathematics to solve the problem. They are encouraged to utilize a word matching method. They are not required to visualize the problem and represent it nonverbally. Table I shows some common phrases and their correspondent operations

**Table I**
**Common Words and Phrases with Their Correspondent Operations**

| Key Word | Operation | Verbal Expression | Alg. Expression |
|---|---|---|---|
| is, are | equal | Courtney is as tall as Ashley. | C = A |
| sum, plus, add to, and, more than, increased by, total | addition | There are six more cars than truck in the parking lot. | C = 6 + T |
| difference, minus, subtracted, from, less than, decreased by, take away | subtraction | The difference between the height of the tallest and shortest student in this class is 4 inches. | T – S = 4 |
| product, times, multiply, twice, of | multiplication | The train took 8 times longer than the plane to get to Chicago. | T = 8P |
| quotient, divide, into, ratio | division | $6000 is ½ of what my car costs. | 6000 = ½ x |



Direct Translation mechanism usually is applicable and valid for majority of simple problems presented to P-8 students. However, for more challenging problems that involves multiple steps of establishing relationships among values rather than quantities, the word matching process is not an effected method due to the occurrence of a phenomenon known by some researchers as Reversal Error. Reversal Error is an empirical phenomenon of translation error pattern.

The following test was administered to a class of 29 in-service teacher students. The students were enrolled in the course Algebra for P-8 Teachers.

1. Write the following sentence as an equation: the number of baseball players (B) is 2 ½ time more than the football players (F) in Carrolton.
2. Write the following sentence as an equation: The children (C) outnumbered the adults (A) by 62 in the Easter egg hunt.
3. Jim invested C= $21000.00 in the stock of his bank last year. The value of his investment has increased at the rate of R = 7.25% since last year. Write an equation to calculate the value of his investment (V) now.
4. The cost of living for a single adult person and in Carrollton was C =18000 last year. The cost of living increses at the rate of R = 4.5% every year in

Georgia. Write an equation and calculate (L) the cost of living in Carrollton this year?

5. Julia bought a car for P=$7000 last year. The value of her car depreciates at the rate of R = 15% each year. Write an equation and calculate the price of Julia's car now?
6. Headwind decreases the normal gas millage of your car by the rate of R = 15%. The normal gas millage of your car is G = 35 mpg. What is the gas millage of your car if you are driving against the wind?
7. Dian receives 4% increase in her annual salary every year. Her salary in 2004 was S = $48650.00. Write an equation and calculate her salary (N) in 2006.
8. After spending C= $2150.00 to replace the roof in his house, John noticed that the new roof costs 25% more than what it cost 15 years before. Write an equation and calculate the amount of the money (P) that he spent on his roof 15 years ago.
9. East Carrollton Motors has found that sales of new cars have decreased 8% from last year. The total sale was C = $6654900.00 this year. Write an equation and calculate (S) the sale last year.
10. After receiving %22 discount we paid P = $40.00 for a pair of shoes. Write an equation and calculate (S) the price of the shoes before the discount.
11. The population of Carrollton increases at the rate of R = 10% each year. The population of the city is N = 35000 peoples. Write an equation and calculate (L) the population 3 year ago.
12. Eight times a number (N) less than 3 is 155. Write an equation and calculate the number.
13. The width (W) of a rectangle is ½ of its length (L). The perimeter of the rectangle is 75m. Write an equation and calculate the width of the triangle.
14. The sum of two numbers is 147 and one number (L) is 6 times larger than the other number (S). Write an equation and calculate each number.
15. The measure of angle B in triangle ABC is 30 degree more than the measure of angle A which is 45 degree less than the measure of angle C. Write an equation and calculate the measure of angles A, B, and C.

An item analysis of the results indicated that:

- First, a majority of the students did not have difficulty answering items 1-4, which were routine algebraic word problems.
- Secondly, some students had more difficulty answering items 5-11 than previous items.
- Thirdly, a significant number of students had difficulty answering items 12-15, which required multiple steps to represent the problems by algebraic expressions.

Item analysis of the results is presented in Table II.

**Table II**
**Item Analysis of the Instrument**

| Item # | Number Correct | Number Incorrect | Percentage Correct | Percentage Incorrect |
|---|---|---|---|---|
| 1 | 28 | 1 | 97% | 3% |
| 2 | 26 | 3 | 89% | 11% |
| 3 | 28 | 1 | 97% | 3% |
| 4 | 25 | 4 | 86% | 14% |
| 5 | 25 | 4 | 86% | 14% |
| 6 | 19 | 10 | 66% | 34% |
| 7 | 18 | 11 | 62% | 38% |
| 8 | 18 | 11 | 62% | 38% |
| 9 | 19 | 10 | 66% | 34% |
| 10 | 18 | 11 | 62% | 38% |
| 11 | 18 | 11 | 62% | 38% |
| 12 | 16 | 13 | 55% | 45% |
| 13 | 16 | 13 | 55% | 45% |
| 14 | 13 | 16 | 45% | 55% |
| 15 | 10 | 19 | 34% | 66% |



## Conclusion

The results indicated the existence of the Reversal Error phenomenon. This phenomenon constituted more than 65% of the wrong answers for items 5-11 and 85% of incorrect answers for items 12-15. The significant drop in the percentage of the correct responds occurred when the problems required expressing the relation of the variable algebraically in more than one step.

Discussing the outcome of the assessment, in an answer and question session after the test, we asked them to draw a picture, figure, or construct a table or graph for each problem and solve them again. The visualization of the problem assisted them to grasp the problems much better. Furthermore, we observed that majority of the students detected their errors and became aware of limitation of direct sentence translation in order to create algebraic formulas to represent the problems. We agree with Fisher, K. (1988) that as mathematics educators we make the wrong assumption that the algebraic notations are self-explanatory for all the word problems. In addition, the findings of the present study are consistent with the findings reported by: Clement, Lochhead, & Monk (1981) for calculus-level students, Fisher (1988) for introductory-level physics students, and Cohen & Kanim (2005) for calculus-based physics students.

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