Understanding the value of a question: An analysis of a lesson

M. Alejandra Sorto, Ph.D.†
Terence Mccabe, Ph. D. ‡
Max Warshauer, Ph. D. §
Hiroko Warshauer ¶

Abstract

Previous research has focused on the characterization of individual questions as to level, type, and pattern (funneling versus focusing). Examining a lesson in context can enhance these characterizations. Using a video and transcript of a geometry lesson, questions are analyzed in isolation, within a local setting, and as part of the entire lesson. These different settings give different interpretations of the value and use of the questions. The appropriate use of questions is one of the most powerful teaching techniques available. By focusing on the big idea of the lesson, the context of the question, and how the question fits into the overall flow of the lesson teachers will be able to understand how to use questions to enhance student understanding and build better lessons. We present a framework for lesson design that teachers can use to make their own questioning more effective.

It takes more than a great question to engage students in mathematical thinking that leads to deep understanding (Boaler and Humphreys 2005). Many articles address questioning by providing theoretical models, analyzing different types and levels of questions, and even by developing a list of "good" questions (NCTM 1991) or a pattern of questions (Herbel-Eisenmann and Breyfogle 2005). As observed by Boaler and Humphreys (2005), it is critical that questions build towards the "key concepts" of the lesson. They also point out that " previous analysis of question types have tended to divide questions rather simplistically into open and closed questions, or higher and lower order questions. But such characterizations do not seem to capture the nuances of the teaching act" (p. 36).

This article extends these ideas to show how one can analyze questions in context to better understand how to use questions effectively in teaching. The suggested framework for analyzing questions takes into account the context and flow of the lesson, while focusing on the underlying mathematical ideas.

Taken in different contexts, the same question can either be appropriate in leading the student to a deeper understanding, or off-key if asked before a student is prepared to understand the mathematical ideas. For example, any attempt to get students to articulate the big idea before they have fully explored the problem and reflected on the process usually results in silence and looks of bewilderment.

In this article we address questioning from a wider point of view. Rather than focus on individual questions, we focus on how these questions play a role in the flow and development of a lesson. Classroom-based examples illustrate how understanding mathematics through questioning depends on the main *goal* of the lesson. By the *goal* of the lesson we do not refer to just an articulation of the specific objectives of a section in a textbook but to understanding an underlying concept or "big idea".

In order to illustrate the ideas above, we examined a geometry lesson conducted as part of a Summer Math Camp for sixth and seventh grade students. First we examined the lesson video and transcript and selected sample questions from the lesson. In the first section below, we classify the questions according to models suggested in previous studies. In the second section, we examine each question in a local context to understand the setting in which the question was posed. In the section that follows, we expand our local analysis to study the questions as a part of the overall lesson. This analysis provides a more complete picture and helps us understand the value of the questions in the teaching context.

Questions in isolation

Consider the following set of questions taken from the geometry class:

1. "How many angles does a rectangle have?"
2. "Does everyone see an angle?"
3. "So does the number 1 plus the number 2 equal 180 degrees?"
4. "So then where did the number 360 come from?"
5. "Does anyone see a pattern here?"

We begin by using already establish models (Table I) to classify these questions by level, type, and pattern.

Question 1 could be classified as a low-level question that is of closed type. Question 2 has elements of being an open type that attempts to probe. Question 4 is of the provoking type, drawing attention to something you want the other person to think about. According to the classification of Martin (2003), Question 5 is the type of question that fosters predicting, inventing, and problem solving. Question 3 does not seem to fit any of the classifications, because it appears that the teacher possibly misspoke. It is almost impossible to accurately determine the intent and the value of each question without knowing the context. Although this is a good start, we can learn more by analyzing where those questions arise within the lesson.

Questions in their neighborhoods=

Question 1 is posed early in the lesson, when the teacher is trying to determine how much students know about angles. This question followed a discussion arising from the previous question: "Where do you see angles?" One of the students responded that he saw angles in triangles. The following conversation resulted:

Teacher:	Are triangles the only shapes that have angles in them?
Students:	Nooo
Teacher:	No? What else does?

Students: Squares.
Teacher: Squares. What else?
Students: Rectangles.
Teacher: Rectangles. What else? Is that the one you wanted to say? [*Looking to a student that put down his hand*] Can you think of another one real quick?
Student: Octagon.
Teacher: Octagons have angles.
Student: Trapezoid.
Teacher: Trapezoids have angles. How many angles does an octagon have?
Students: Eight.
Teacher: That's exactly right. How many angles does a rectangle have?
Students: Four.
Teacher: Four. How many angles does a trapezoid have?
Students: Four.
Teacher: So, what's the relationship then between the angles and the shape?
Student: If it has 4 sides, then it has 4 angles.
Teacher: You got it.

After this discussion, the teacher quickly moved to the next question, "Whose jobs might involve things with angles?" In this part of the lesson, questioning is fast paced (less than one minute) and clearly establishes the students' background knowledge about angles. The teacher used the student's responses to establish a relationship between angles and polygons through a fast sequence of closed questions. This pattern of questions is of the kind of gathering information where the teacher wants direct answers, usually right or wrong rehearsing known facts (Boaler and Humphreys 2005), leading towards a mathematical relationship. Notice how the teacher is satisfied with the response being generalized only for quadrilaterals and not for all polygons ("If it has 4 sides, then it has 4 angles"), giving an indication that this was not a major part of the lesson. Hence, Question 1 and the pattern of questions around it are closed and lower-level type, with the intention of leading towards a mathematical relationship. Because of the fast paced nature of the questions and the fact that this particular conversation is embedded in the conversation about angles in general, the mathematical relationship is not deeply developed and it is not clear if students are grasping the connections between angles and shapes. At this point of analysis, we understand better why certain low level questions are asked and we can value the question in the context of guiding students to see a general mathematical relationship.

In contrast, question 5 "Does anyone see a pattern here?" occurs much later in the lesson. When we look at the context, we find out that this question is not used to predict what would come next in a pattern as suggested in the first classification of the question. The teacher is using this question to bring up the idea of precision when measuring angles, as illustrated in the following discussion:

Teacher: So, using your protractor when you guys measured angle 1, can anybody tell me the measure that they found? Alex?
Alex: 135.
Teacher: 135°. Do ya'll agree?
Teacher: I'm seeing some hmm faces. What did you get?
Kyla: I got 137.
Teacher: Okay, so let's do this. Alex says it's 135, Kyla says it's 137, did anybody else get something different?
Student: 134.
Teacher: 134, [*Elliot raises his hand*] I'm sorry I don't think I ever caught your name. What's your name?
Elliot: Elliot.
Teacher: Elliot, what did you get?
Elliot: 136.
Teacher: 136. This is interesting to me. Does anyone see a pattern here?
Students: They are off by one
Teacher: Yeah, they're all only off by one.

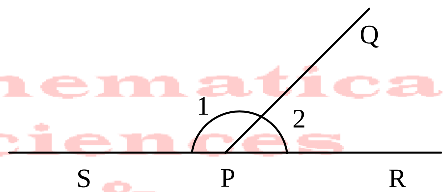
Because the goal of this conversation was not to predict what would come next in a pattern but to notice the variation of the measurements, we initially thought that this was not an appropriate question to ask. So, we might value this question in the context of students trying to make predictions or generalizations about numerical or geometrical patterns but not in the context that the teacher used here. A more appropriate question could have been "Why are we getting different measurements for the same angle?"

Question 4, "So then where did the number 360 come from?" was also a provoking and interesting question. Before examining this question in context, we would have thought that this question was in the main part of the lesson. However, we later understood that the teacher chose to pose it not with the intention of getting immediate answers but rather to generate curiosity about a mathematical fact. She asked "Why are there 360 degrees in a full circle?", created some discussion, and finally asked the for students to explore this question further on their own. This question was posed at the beginning of the lesson as part of the initial background.

Question 3 is actually the heart of the lesson. It is part of a complete discussion about using appropriate language when deriving an equation that involves angles, degrees and measure:

Teacher: Okay. So now what I would like for you guys to do is tell me what happened when we smooshed those two angles together?
Students: We got 180°.

Teacher: We got 180°, does everybody agree with that? All right, so this is what we're going to do: we're going to call this one angle 1 because that's just a shorter name for it, and we're going to call this one angle 2 just because it's a shorter name for it. And I want you guys to help me write an equation based on what Ms. Kyla shared with us. She said that when we smooshed those together they made 180 degrees. You said you all agreed, so I want to see an equation that says the same thing.



Student: 1 plus 2 equals 180°.

[*Teacher writes on the board* 1 + 2 = 180° *leaving plenty of space between the plus sign and the number 2*]

Teacher: Notice that Alex did not say 1 plus 2 equals 180, because if we all walked out of here today and we all told our parents that 1 plus 2 equals 180, what would they say to us? Quickly get out of this crazy lady's class, because 1 plus 2 would never equal 180. So he said something very important; he said it equals 180°.

So does the number 1 plus the number 2 equal 180°?

No, no no no no, but you're on the right track. So, let's expand. What we have right now is a beginning, there's nothing wrong we're just not finished yet. So who can help me expand a little bit?

Teacher: Okay, here's the symbol for angle, angle 1 plus angle 2 equals 180°.

[*The teacher now writes the symbol for angle next to the number 1 and number 2 and now she has the following on the board* $\angle 1 + \quad \angle 2 = 180^\circ$]

I want to expand on what Kyla just said though, because the only way that you can end up with an answer that is in degrees is if you start off by adding degrees together. So what am I asking you to do to that angle 1in order for it to represent degrees?

Student: Find how big it is.

Teacher: Find how big it is. So what am I asking you to physically do to find out how big it is?

Students: Turn it into degrees.

Teacher: Turn it into degrees. And what did we just discover, how to we turn an angle into degrees?

Students: You measure it.

Teacher: You measure it, excellent. You measure it with a protractor. So the way we write this is equation is we say not just angle 1, but the *measure* of angle 1. Notice how great math is, because we don't have to write all that junk out. Measure, what did I just turn it into?

Students: *m*.

Teacher: *m*. I took that big long name QPS and I turned it into 1, I didn't have to write the word angle, I draw that little symbol. So the equation is: the measure of angle 1 plus…what do I need to stick here?

Students: *m*

Teacher: The *measure* of angle 2… equals 180°.

[*The teacher now completes the equation* $m\angle 1 + m\angle 2 = 180°$]

We can see now that the teacher did not misspeak. Her question fell in the middle of the discussion and was actually a reflection of one of the student's responses. She made the point that we need to be very careful about the way we use our words to precisely describe angles, measure and degrees.

The teacher in this part of the lesson used questioning in a masterful way to develop the idea of relating measures of angles to mathematical equations. Notice the way that she values the students' responses, and then builds on their prior knowledge.

Questions in the overall lesson

By examining the entire lesson, we can better understand how the teacher planned the lesson because most of her questions have a carefully defined role. Figure I illustrates the entire flow of the lesson with eight different parts (Part I – VIII). The identification of the different parts was obtained after transcribing the entire videotape of the lesson and mapping the type of questions to the framework suggested by Boaler and Humphreys (2005). Questions are posed in different formats (whole class discussion and group work) according to the goal of each part of the lesson. The questions in Figure 1 are examples of teachers' questions that the teacher planned in advance. Many more questions were asked that arose as a consequence of students' responses, as previous transcripts illustrate.

As a whole, the lesson was structured to lead students to master a procedure (Part IV) and to explore mathematical meaning and relationships (Part V and VI). These correspond to the two major objectives of the lesson: measuring angles and exploring supplementary angles, which were rapidly communicated at the beginning of the lesson (Part I). We did not observe questions that allow students to explicitly explain their thinking or elaborate their thinking for their own benefit and for the class. The lesson and questions can now been placed into a structured framework.

VI. Exploring mathematical relationships

After the short introduction, the teacher generates a whole class discussion with the intent of establishing context around angles (Part II). She poses a sequence of questions with the specific goal of gauging student's prior knowledge about angles and relating the abstract concept of angles with real applications. In addition to asking the pre-planned questions listed in Part II of Figure 1, the teacher asked questions that generated discussion such as "Does anyone disagree with him?" "Why would that be important in an angle?"

We notice two deviations from this sequence that arise from students' responses to the questions posed. Evidence for these deviations in the discussion was the little time the teacher spent and the refocus on the original question "What do you know about angles?" The first deviation originated when the teacher asked "Does that make anybody else think of something else?" and one student responded, "The biggest angle is 180 degrees". A one minute and 7 seconds discussion about this fact followed, indicating that the teacher did not want to spend too much time on this issue. The second deviation was when students said that they see angles in shapes. This prompted the teacher ask another sequence of questions leading to "What's the relationship then between the angle and the shape?" Question 1, "How many angles does a rectangle have?" was part of this sequence. This discussion only lasted about 35 seconds. As the timing suggests, the pace of questioning in this part of the lesson was quite quick, indicating that this was just a discussion to build a common landscape of prior knowledge without going into detail on any single issue.

In Part III, the teacher revisits the objectives of the lesson, summarizes the previous discussion, and signals a shift in the focus of the lesson. There are no questions asked in this part of the lesson. The flow of the lesson changes in Part IV as students experience the measuring of angles in a group setting. As they prepare to make these measurements, the teacher poses another sequence of questions with the purpose of leading the students to focus on what is needed in order to make and measure an angle. The teacher uses the discussion about what is needed to introduce appropriate terminology such as ray, vertex, and straight edge. This set of questions does not require students to make connections to applications like questions in Part II but rather rehearses known procedures. This enables the students to talk about these concepts using correct mathematical terminology. Because the purpose in this part is not to establish the context, the pace of the questions slowed as the students become actively involved in the procedure of measuring at the same time that they are answering questions.

As the teacher brings the students together after the group work, she uses this opportunity to talk about the nature of the variation of measurements by posing the following question "Were people getting answers either the same or really really close to what you got?" Again, the teacher does not take too much time here (less than a minute). The intent is simply to cause the students to observe that when we measure, there will be variation.

Parts V and VI are at the heart of the lesson. Questions in these parts are intended to explore mathematical meanings and relationships. Boaler and Humphreys (2005) claim that these are the most important type of questions as they orient the students to the central mathematical idea. "They do not necessarily follow up on students' ideas; they often come from the teacher, and they serve a very particular and deliberate purpose: challenging students to consider a critical mathematical concept" (p. 38).

Part V begins an exploration of how one can express a mathematical relationship as an equation using symbols. This is introduced by Question 2 "Does everyone see an angle?" Because it occurs right before the development of the equation for supplementary angles, we see that the purpose of Question 2 is to assess whether students have identified the two angles the teacher intends to use. Although the conversation is short (about 2 minutes), students get to see and explicitly name the two angles using letters. Question 3 is found in this part of the lesson as well, where the teacher enables her students to articulate the equation. Here is where the "big idea" of the lesson is being addressed: the concept of angle, its measure and the precise mathematical way of expressing the relationship between supplementary angles. By the time spent in this part of the lesson (over 4 minutes) in relation to the other sections; there is evidence that the teacher paced these questions slowly and carefully.

Part VI is dedicated to discussing the relationship formally established in the previous conversation. The teacher asks students to give possible values for angle 1 and angle 2 that would make the equation true, concluding that "Okay, there's lots of different things it could be." This exploration gives students the opportunity to think of " $m\angle 1$" as a variable and to experience an equation with multiple solutions. The teacher and students derive the formula with a specific example and students can find out the actual values of the measures of angle 1 and angle 2. At this point the teacher wants students to convince themselves that the equation that they derived "works" by measuring the angles and evaluating the equation. The teacher says "So, there without measuring we could pick any old thing to be 180 degrees, not any old thing but quite a few things. So what I want to know is what is the measure of angle 1, and what is the measure of angle 2. So what you think you need to do?" Students all agreed that they needed to measure the angles. As each student practices their measuring skills, the teacher is now ready to begin the next discussion about the variation and precision of measurements by posing the question "Can anybody tell me the measure that they found?" Students respond by giving their measurements of angle 1 and the teacher writes these responses on the board. There is some variation for the values of "$m\angle 1 \quad m\angle 1$." The teacher leads a discussion of this variation by posing Question 5 "Does anyone see a pattern here?" We now see again that this question was not fostering predictions as we first thought. She only intended for the students to notice the variation in measurements when we use an instrument such as a protractor.

She then asks another interesting question about the measure of angle 2 "That means should we all have the same thing for measure of angle 2?" As students measure angle 2 they realized that the measurements also vary, however their sum did not. Although questions here were asked to explore mathematical relationships, there was not sufficient evidence that the students were actually understanding these relationships. Their short answers refer to the arithmetic of adding two numbers and checking if they add to 180. The questioning in this part of the lesson was also quite slow as the teacher spends another 4 minutes exploring the relationships. The flow of the lesson changes as students work individually on what the teacher calls “The Challenge” which consists on creating an equation for four angles whose measure sums to 180 degrees. The teacher finishes the lesson by summarizing the main points.

Implication for teaching

In the daily work of teaching mathematics, questioning is a powerful tool that teachers can employ. The use of questioning dates back to Socrates, and much of the research that followed has focused on how one can analyze the value of a question. Teachers often seek resources (e.g. teacher’s guide) that provide lists of questions they should ask. Because these questions have been listed in isolation, they often are not effective.

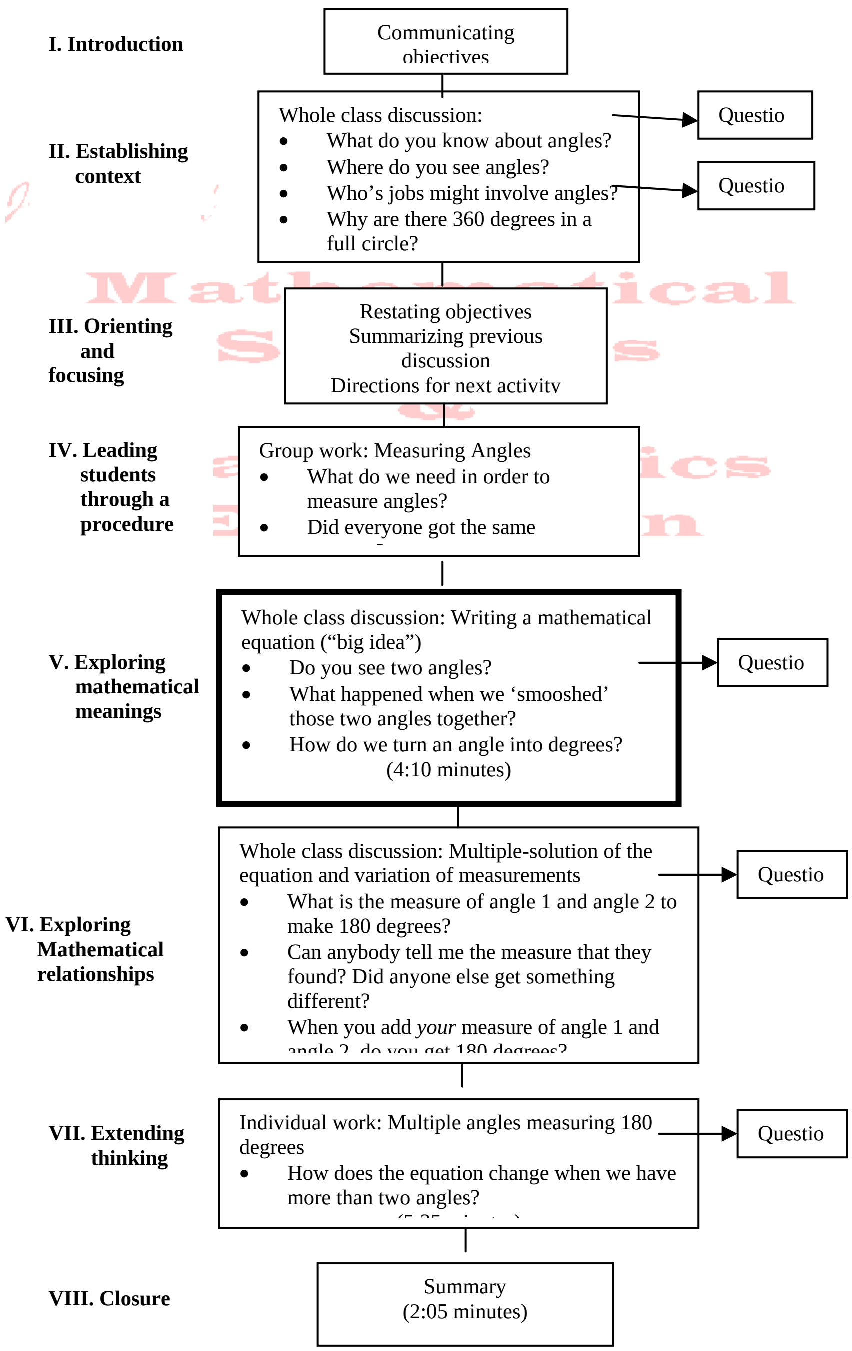
Our analysis of a sample lesson used a framework for analyzing questions in context, with questions occurring in one of 8 sections. Questions derive their value from their context and flow in the lesson, with a focus on student understanding of the big idea.

There is no formula for knowing what to ask and when, but there are basic principles that one can follow. The examples given show how questions can make a lesson come alive. Questions can be used for many purposes, as seen in the lesson we studied. Much of the joy of teaching comes from allowing students to explore ideas for themselves, and this exploration can be guided by thoughtful questions that lead to deeper understanding. So when weaving in questioning into a lesson, let the big idea drive the questions, not the other way around!

Table I
Classification of questions by level, type, and patterns.

Level	Cognitive Higher order vs. Lower Order (Wimer, et al. 2001)	*Examples:* *Why did that work in this case?* *What is the special name of the triangle?*
Type	Open vs. Closed Genuine Provoking Foster predicting, inventing Empowering (Martin 2003)	*Examples:* *How many different triangles did you find?* *Is there a pattern? What is it?* *Why is that?* *Did anyone do it a different way?*
Patterns	Probing Orienting Funneling Focusing (Herbel-Eisenmann and Breyfogle 2005, Boaler and Humphreys 2005)	*Examples:* *How did you get 10?* *Would this work with other numbers?* *What is the problem asking you?* *What is important about this?* *Then you want to do what?*

Figure I
The flow of the lesson



† *M. Alejandra Sorto, Ph.D.*, Texas State University-San Marcos, Texas, USA
‡ *Terence Mccabe, Ph. D.*, Texas State University-San Marcos, Texas, USA
§ *Max warshauer, Ph. D.*, Texas State University-San Marcos, Texas, USA
¶ *hiroko warshauer*, Texas State University-San Marcos, Texas, USA

References

Boaler, Jo and Cathy Humphreys. (2005). *Connecting Mathematical Ideas: MiddleSchool Video Cases to Support Teaching and Learning*. Portsmouth, NH: Heinemann.

Herbel-Eisenmann, Beth and M. Lynn Breyfogle. "Questioning Our Patternsof Questioning." *Mathematics Teaching in the Middle School* 10 (May 2005): 284- 289.

Martin, Nikki. "Questioning Styles." *Mathematics Teaching* 184 (September 2003): 18-19.

National Council of Teachers of Mathematics ,NCTM, (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM,

Wimer, J., el al. (2001), "Higher Order Teacher Questioningof Boys and Girls in Elementary Mathematics Classrooms". *The Journal of Educational Research* 95(2): 84-92.