Random Devices Utilized in Mathematics Textbooks

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Abstract

This paper describes the results of a study of the probability content of middle grades (6-8) mathematics textbooks published in the U.S. between the 1960s and 1990s. In particular, an analysis of the types of devices used in probability tasks is presented. Across all textbook series, most probability tasks were set within the context of using a random device. The most common types of devices used involved selecting an object at random, cubic dice, coins, or spinners. The largest number of different types of devices occurred in the two most recently published series; these series also offered the only tasks where devices were used to model other phenomena.

Introduction

Misconceptions about probability are both widespread and persistent. This is documented in the research literature (e.g., Kahneman, Slovic, & Tversky, 1982; Konold, 1983; LeCoutre, 1992; Shaugnessy, 1992, 2003) and informally noted by teachers of mathematics and statistics. For example, students may believe that all of the possible outcomes of an experiment are equally likely, regardless of the situation. LeCoutre (1992) referred to this as the equiprobability misconception. To help students develop an understanding of chance and randomness, the National Council of Teachers of Mathematics (NCTM, 2000) has recommended connecting the study of probability to data analysis, and use simulations to model real-world phenomena. These recommendations are reiterated and endorsed by the American Statistical Association in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report* (Franklin et al., 2007).

The emphasis on probability in the school mathematics curriculum has increased over the past century (Jones, 2004), from being reserved for college-bound high-school seniors in 1959 (College Entrance Examination Board, 1959) to a topic for *all* students to study, beginning as early as the elementary grades (NCTM, 1989, 2000). While it was recommended that probability receive increased attention over the past years, there is still a question of how these recommendations were implemented into actual classrooms. Mathematics textbooks are an important component in the implementation process, as they are a teachers' primary resource (Grouws & Smith, 2000; Tyson-Bernstein & Woodward, 1991), and they provide a snapshot of mathematics instruction at a particular time.

In light of the emphasis on using simulations to examine events involving chance, I decided to investigate the random devices (e.g. coins, spinners, etc.) that were included in the probability portions of middle-grades (grades 6, 7, and 8) mathematics textbooks published in the U.S. between the

1960s and the 1990s. Specifically, the research questions were: (1) What devices are used in probability tasks? (2) Are students asked to study the properties of the device, or use the device to model some other phenomenon? (3) What are the trends in the type and nature of device over the four most recent eras of mathematics education?

Sample Selection and Methodology

In order to determine historical trends in the treatment of probability in curricular materials, I selected two textbook series from four *recent eras of mathematics education* (Fey & Graeber, 2003; Payne, 2003): the New Math, Back to Basics, a focus on Problem Solving, and the advent of the National Council of Mathematics' [NCTM] *Standards* documents (NCTM, 1989, 2000). It is difficult to determine the precise beginning and end of these eras, and a significant event that marks the start of a new era (e.g., the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989) does not necessarily immediately impact the textbooks that are published that year or the next. Nevertheless, I acknowledge the need to specify time frames for each era. I used the following operational time frames for the four eras: New Math (1957-1972), Back to Basics (1973-1983), Problem Solving (1984-1993), and Standards (1994-2004).

For each era, I selected two series of mathematics textbooks. The first series, *popular*, was a series that was used by the largest proportion of middle-grade students in the United States, as determined by market share data. The second series, denoted *alternative*, was different from *popular* textbooks at the time, possibly because of the authors' desire to reform mathematics education by providing alternative curricular materials. The alternative series were identified through a "professional consensus" of mathematics educators affiliated with the Center for the Study of Mathematics Curriculum and familiar with the mathematics curricula of the eras included in this study. I examined both popular and alternative textbook series from each era in an attempt to gain a broad perspective on the treatment of probability topics for that era. Table I lists the 24 textbooks that were included in the sample for this study.

I examined each page of the selected textbooks to identify the *probability tasks* contained therein. Drawing heavily on the work of the QUASAR Project (e.g., Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998; Stein, Smith, Henningsen, & Silver, 2000), I use the term *probability task* (or simply *task*) to refer to an activity, exercise, or set of exercises in a textbook that has been written with the intent of focusing a student's attention on a particular idea from probability. Any task that contained probability was considered a probability task, even if the main focus of the task was on another content area, such as geometry, combinatorics, or statistics.

Table I
Textbook series analyzed

Era	Type	Textbook Titles & Publisher	Publisher
New Math	Popular	*Modern School Mathematics: Structure and Use 6, Modern School Mathematics: Structure and Method 7 & 8*	Houghton Mifflin
	Alternative	*Mathematics for the Elementary School, Grade 6 Mathematics for Junior High School, Vols. I & II*	Yale University Press
Back to Basics	Popular	*Holt School Mathematics: Grades 6, 7, & 8*	Holt, Rinehart, & Winston
	Alternative	*Real Math: Levels 6, 7, & 8*	Open Court
Problem Solving	Popular	*Mathematics Today: Levels 6, 7, & 8*	Harcourt Brace Jovanovich
	Alternative	*Math 65: An Incremental Approach Math 76: An Incremental Approach Math 87: An Incremental Approach*	Saxon Publishers
Standards	Popular	*Mathematics: Applications and Connections: Courses 1, 2, & 3*	Glencoe/ McGraw-Hill
	Alternative	*Connected Mathematics*	Dale Seymour

A probability task is not necessarily a single exercise in the textbook. A set of exercises that build on one another is considered as a single task. I have constructed such a task, as illustrated in Figure I. Likewise, a set of exercises that attend to the same topic but may be answered in isolation is considered as one task, as is the case in the task I constructed for Figure II. Sections of probability lessons that contain narrative, such as definitions or written explanations of concepts and procedures, were not considered as probability tasks. Most probability tasks were located within lessons, in both the development (e.g., worked examples, activities) and assignment portions of lessons. Other probability tasks were not located in lessons, but in chapter reviews, assessments, and extension or enrichment activities.

Figure I
Probability task with multiple related questions

How likely is it that a chocolate chip will land on the flat side after being tossed in the air? Perform the following experiment and answer these questions to help formulate your answer to this question.

1. What are the possible outcomes for the landing position of a chocolate chip?
2. With your partner, toss 50 chocolate chips and record the landing position. How many chips landed on the flat side?
3. Based on your data, what is the experimental probability of a chocolate chip landing on the flat side?
4. As a class, pool your data. Based on the pooled data, what is the experimental probability of a chocolate chip landing on the flat side?
5. How does the experimental probability based on your data compare to the experimental probability based on the pooled data? How do you account for any differences?
6. Which of these experimental probabilities do you believe to be closest to the theoretical probability? Why? How could you obtain a better estimate of the theoretical probability?

Figure II
Probability task with questions that could be answered in isolation

A candy dish contains four red lollipops, five blue lollipops, and six green lollipops. Lucy selects one lollipop from the dish at random.

1. What is the probability that the lollipop is red?
2. What is the probability that the lollipop is yellow?
3. What is the probability that the lollipop is not blue?

I examined each task according to the device(s) that were incorporated into that task to establish a probabilistic situation. Examples of such devices were coins; spinners; selecting marbles in a jar, letters in words, or items from a menu; and using computer programs. I recorded the device(s) used in each task, and calculated the frequency of each type of device for each series. I also coded each task in terms of the purpose of the device:

- Device-Model – Students are asked to use a device to model some other probabilistic situation. A task that requires flipping two coins 50 times to model the sexes of 50 pairs of siblings was coded as Device-Model.
- Device-Reflexive – Students are asked to study the properties of the device itself. For example, the tasks in Figure I and Figure II would both be coded as Device-Reflexive.
- No Device – In the event that a task does not reference any type of device, the task was coded as having No Device. Such tasks are based solely on text and mathematical reasoning, such as, "The probability of event A occurring is 0.4, and the probability of event B occurring is 0.2. Assuming that these events are independent, what is the probability of both events occurring?"

It was possible that a task referred to more than one device. Within each of these tasks, the nature of the device (reflexive or modeling) was consistent for all devices utilized within the task. I calculated the percentage of probability tasks within a series that were coded as Device-Model, Device-Reflexive, and No Device.

Results

Tasks Using Devices

The number of probability tasks within each series is shown in Table II. For five of the eight series, there are roughly 100 probability tasks contained in the textbooks intended for grades 6, 7, and 8 combined. This stands in contrast to the textbooks from the Standards era, which nearly contained that number in each grade-level textbook in the series. The Problem Solving-Alternative series devoted the least amount of attention to probability with only 42 tasks spread across the three grade levels.

Table II
Probability tasks using devices

	New Math		Back to Basics		Problem Solving		Standards	
	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.
Number of probability tasks	113	85	85	107	103	42	331	216
Percent of probability tasks using devices	62	65	93	79	83	86	67	70

The majority of tasks in each series incorporated the use (or the suggestion of use) of some sort of device. The proportion of probability tasks within a series that used a device ranged from 62% in the New Math-Popular series to 93% in the Back to Basics-Popular series. While 67% and 70% of probability tasks in the two Standards-era series used devices, it should be noted that the actual number of tasks using devices in these series is greater than the total number of probability tasks from any series in the other eras.

Types of Devices

Across the eras, the number of types of devices used in tasks grew from the New Math era to the Standards era. The most common device for several eras was the selection of an object at random, such as a marble from a jar or a slip of paper from a hat. Table 3 displays the percent of probability tasks in each series that used devices of a particular type. Four types of devices were used in almost every series: the selection of an object at random, cubic dice, coins, and spinners. The types of devices remained relatively stable across the eight series, although the proportion of tasks utilizing a particular device varied from series to series. For example, the New Math-Popular referred to selecting some sort of object at random in 81% of the probability tasks that used devices. By way of contrast, in the Back to Basics-Alternative series, only 7% of such tasks referred to selecting an object; dice were used most frequently in this series. Two series (New Math-Popular and Back to Basics-Alternative) did not use spinners in any probability tasks, while spinners were the most frequently used device in the Problem Solving-Alternative series.

Table III
Percent of probability tasks using various types of devices

Series / Device	New Math		Back to Basics		Problem Solving		Standards	
	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.	Pop.	Alt.
Select Object	81	46	30	7	34	3	33	21
Cubic Dice	12	13	31	43	16	22	16	21
Coin	6	28	20	30	34	14	15	16
Spinner	0	10	17	0	26	35	14	19
Other	0	3	2	20	0	0	23	22

Note. Percents in each series may not sum to 100 due to rounding.

The devices represented in the "other" category for the tasks in the New Math-Alternative and Back to Basics-Popular series were exclusively regular (non-cubic) polyhedral dice, such as tetrahedra, octahedra, dodecahdra, or icosahedra. One task in the Back to Basics-Popular series also used a pentahedron, described as a solid with five faces. Within the Back to Basics-Alternative, Standards-Popular and Standards-Alternative series, the "other" devices included the use of area-based devices such as dartboards or maps in which the calculation of areas facilitated the calculation of probabilities. The remaining devices classified as "other" in the Standards-Popular series were computer programs, tossing objects that would not necessarily yield equally likely outcomes (e.g., cups or tacks), and balls falling through a specially designed chute. For the Standards-Alternative series, the remaining devices classified as "other" were tossing objects that would not necessarily yield equally likely outcomes (e.g., marshmallows or bottles) and random number generators. Furthermore, both series from the Standards era contained tasks in which students selected or created a device to model some phenomenon; that is to say, the type of device used depended on the student's selection.

Nature of Devices

During the New Math, Back to Basics, and Problem Solving eras, all devices were used reflexively. That is to say, students were asked to analyze the properties of the device, as in the tasks shown in Figures I and II. Within the Standards-Popular series, 14% of probability tasks with devices were used to model some other phenomenon. For the Standards-Alternative series, this proportion was 13%. Figure III contains an example from the eighth grade textbook of the Standards-Popular series (Collins et al., 1998) demonstrating how a computer program is used to model a free-throw situation in the game of basketball.

Figure III
Probability task where a device is used to model a phenomenon (From Collins et al., 1998, p. 530)

In the "bonus situation" in basketball, the shooter is awarded one free throw. If he or she makes the basket, a bonus of one more free throw is awarded. Therefore, there are three possibilities:

A. Shooter misses first free throw.
B. Shooter makes first free throw, misses the second.
C. Shooter makes both free throws.

Suppose the probability that a certain shooter makes any given free throw is 67%. The following computer program simulates the results of 10,000 trials and prints the experimental probability of each situation.

```
10  FOR X = 1 TO 10000
20  LET Y = RND(X)
30  IF Y > 0.67 THEN CA = CA + 1: GOTO 70
40  LET Y = RND(X)
50  IF Y > 0.67 THEN CB = CB + 1: GOTO 70
60  CC = CC + 1
70  NEXT X
80  PRINT "P(A) ="; CA/10000: PRINT "P(B) ="; CB/10000: PRINT "P(C)
    ="; CC/10000
```

a. Run the program and list the probability of each outcome.
b. Modify the program if the probability that the shooter makes any given basket is 50%.

Discussion

For the past several decades, professional organizations such as the NCTM (1980, 1989, 2000) have recommended that students have opportunities to connect mathematics to the world around them. The results of this study document that this recommendation has yet to be fully actualized within textbooks, at least in the area of probability. Across the series, there was a predominance of probability tasks that were either non-contextual or situated in an artificial context, such as selecting a sock from a drawer without looking. Every device from the New Math, Back to Basics, and Problem Solving eras, and nearly 6 out of 7 devices in the Standards era was used reflexively (e.g., determining the probability of getting three "heads" when tossing three coins). This overuse of Device-Reflexive tasks further reveals the artificial contexts of many probability tasks.

In terms of the nature of devices, both series from the Standards era used some devices to model other phenomena. No other series used devices in this way. The emergence of using devices to model phenomena in the Standards era coincides with recent recommendations to utilize modeling to study probability (e.g., NCTM 1989, 2000; Shaughnessy, 1992, 2003). Additionally, the series from the Standards era utilized devices that may not necessarily yield equally likely outcomes. Using such devices (e.g., spinners with sectors of unequal areas, tossing thumbtacks or chess pawns) may help to address and confront the misconception that all events within the sample space of an experiment are equally likely (Bright, Frierson, Tarr, & Thomas, 2003; LeCoutre, 1992).

The findings reported here are part of a larger study (Jones, 2004), which revealed that, with the exception of the Back to Basics-Alternative series, one task in eight (or fewer) addressed experimental probability during the New Math, Back to Basics, and Problem Solving eras. By way of contrast, during the Standards era nearly 30% of tasks addressed this topic (Jones, 2008). Both textbook series from the Standards era were also the only two series to use devices to model other phenomena. These results support the recommendations of researchers (e.g., Aspinwall & Tarr, 2001; Shaughnessy, 1992; Shaughnessy, Canada, & Ciancetta, 2003; Stohl & Tarr, 2002) that students should be afforded opportunities to study experimental probability, particularly through modeling. Moreover, instruction in probability should help students forge links between theoretical probability and experimental probability as recommended by Shaughnessy et al. (2003):

> All too often we rush our students to calculating the probability of individual events or probabilities of particular outcomes, without consideration for the variation in results that can occur in actual repeated trials. We rarely give our students opportunities to develop their intuition for a likely "range of outcomes" in repeated trials situations, especially when there is a convenient probability model … to tap. … It is not just the exact probability that is important in data and

> chance, but perhaps even more so, how that outcome is situated within the distribution of outcomes for an experiment, and what the "likely range" of outcomes of the experiment will be. (p. 164)

Therefore, probability tasks within textbooks should involve the modeling of real-world phenomena through simulations, and in doing so, promote key connections between data and chance.

Conclusion

Over the past half-century, probability has moved from a topic that was reserved for the most-able high school seniors to one that is studied by *all* students, beginning in elementary school. With this growth have come recommendations for connecting probability (along with other areas of mathematics) to real-world phenomena. This can best be accomplished in the classroom by modeling such events with random devices. Because the real world contains a wide variety of phenomena to investigate, careful attention should be given to include both devices that yield equally likely outcomes and devices that yield outcomes with different probabilities. While it is important to analyze particular random devices, it is also important for students to see how probability can be used as a tool for statistics in modeling random events, as promoted by the GAISE (Franklin et al., 2007). Experience in using a variety of devices, both in analyzing the device and utilizing a device to model another situation, will afford students the opportunities necessary confront and address possible misconceptions, and ultimately increase their understanding of probability.

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References

Aspinwall, L., & Tarr, J. E. (2001). Middle school students' understanding of the role sample size plays in experimental probability. *Journal of Mathematical Behavior, 20*, 229-245.

Bright, G. W., Frierson, D., Jr., Tarr, J. E., & Thomas, C. (2003). *Navigating through probability in grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

College Entrance Examination Board. (1959). *Program for college preparatory mathematics*. New York: Author.

Collins, W., Dristas, L., Frey-Mason, P., Howard, A. C., McClain, K., Molina, D. D., et al. (1998). *Mathematics: Applications and connections, course 3*. New York: Glencoe/McGraw Hill.

Fey, J. T., & Graeber, A. O. (2003). From the new math to the *Agenda for Action*. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 521-558). Reston, VA: National Council of Teachers of Mathematics.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework.* Alexandria, Va.: American Statistical Association.

Grouws, D. A., & Smith, M. S. (2000). Findings from NAEP on the preparation and practices of mathematics teachers. In E. A. Silver & P. A. Kenney (Eds.), *Results from the Seventh Mathematics Assessment of the National Assessment of Education Progress* (pp. 107-141). Reston, VA: National Council of Teachers of Mathematics.

Jones, D. L. (2004). *Probability in middle-grades mathematics textbooks: An examination of historical trends, 1957-2004.* Unpublished doctoral dissertation, University of Missouri-Columbia.

Jones, D. L. (2008). Probability topics addressed in U.S. middle-grades textbooks, 1957-2004. *International Journal for the History of Mathematics Education, 3*(1), 1-18.

Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment under uncertainty: Heuristics and biases.* Cambridge: Cambridge University Press.

Konold, C. (1983). Conceptions about probability: Reality between a rock and a hard place. (Doctoral dissertation, University of Massachusetts, 1983). *Dissertation Abstracts International, 43*, 4179b.

Lecoutre, M. (1992). Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics, 23*, 557-568.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Payne, J. N. (2003). The new math and its aftermath, grades K-8. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 559-598). Reston, VA: National Council of Teachers of Mathematics.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494). Reston, VA: National Council of Teachers of Mathematics.

Shaughnessy, J. M. (2003). Research on students' understandings of probability. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 216-226). Reston, VA: National Council of Teachers of Mathematics.

Shaughnessy, J. M., Canada, D., & Ciancetta, M. (2003). Middle school students' thinking about variability in repeated trials: A cross-task comparison. In N. A. Pateman, B. J. Dougherty & J. T. Zilliox (Eds.), *Proceedings of the 2003 Joint Meeting of PME and PMENA* (Vol. 4, pp. 159-165). Honolulu, HI: University of Hawai'i.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*, 344-350.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection. *Mathematics Teaching in the Middle School, 3*, 268-275.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Stohl, H., & Tarr, J. E. (2002). Developing notions of inference using probability simulation tools. *Journal of Mathematical Behavior, 21*, 319-337.

Tyson-Bernstein, H., & Woodward, A. (1991). Ninteenth century policies for twenty-first century practice: The textbook reform dilemma. In P. G. Altbach, G. P. Kelly, H. G. Petrie & L. Weis (Eds.), *Textbooks in American society: Politics, policy, and pedagogy*. Albany, NY: State University of New York Press.