Prospective Middle Grades Teachers of Mathematics Using Hand-Held CAS Technology to Create Rich Mathematical Task

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Abstract

The purpose of the study was to examine the effectiveness of using a hand-held Computer Algebra System (CAS) to create rich mathematical explorations for middle grades students. Participants were 20 undergraduate students. They used a CAS in a mathematics education content course designed to make connections to the middle grades for such topics as number theory, trigonometry, and calculus. The results of the study demonstrated that the prospective teachers were able to design a rich mathematical activity using CAS. The prospective teachers' teaching philosophy for using CAS with middle grades students was changed.

Background

In earlier discussions, the hand-held Computer Algebra System (CAS) was thought of as a hindrance to school mathematics. This debate sparked researchers to examine the influence of CAS on student conceptual understanding of mathematics. Through this research, CAS technology to aid in the teaching and learning of mathematics at the secondary and collegiate levels was proven beneficial (Bossé & Nandakumar, 2004; Day, 1993; Heid, 1988; Palmiter, 1991; Vlachos & Kehagias, 2000). The use of CAS technology is not typically used with middle grades students, although the capabilities could enhance student learning. NCTM's *Principles and Standards for School Mathematics* indicates that "technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives" (p. 25). It is our attempt to examine middle grades prospective teachers (PTs) of mathematics using a CAS device to create rich mathematical explorations.

After an extensive search of the literature, no research studies or theoretical perspectives describe middle grades teachers, students, or prospective teachers using CAS to make sense of mathematics, although technological tools have encouraged a pedagogical shift at the K-12 levels. In the past decade, four-function calculators were a part of the curriculum at the elementary level; however, with the advancement of technology, elementary teachers find themselves utilizing two-line calculators. Two-line calculators have the capability to display expressions on one line and solutions on the second line (St. John and Lapp, 2000). Additionally, middle grades teachers of mathematics were reliant on scientific calculators, however with the change moving to graphing calculators; many middle grades teachers are implementing graphing calculators. Numerous studies indicating the importance of the graphing calculator as it transcends what middle grades teachers do in the classroom (Handheld Graphing Technology Facilitates Learning, 2001; Heid & Edwards, 2001; Oldknow, 2001;).

The case we are making is with the evolution of technological tools to aid middle grades teachers; teachers also find themselves transforming what they do in the classroom. Similar to what Fennema and Franke (1992) describe what teachers know about mathematics is an important factor for what they do in the classroom. We argue that the same can be said for what teachers know about the effectiveness for teaching with technology is an important factor for what they do in the classroom. As well, Zbiek and Hollebrands (2008) support the view that "Teachers' conceptions, beliefs, knowledge, and use of technology influence the activities they create for their students" (p. 310).

Framework

This research focuses on PTs content knowledge and their use of CAS to create rich mathematical explorations. An earlier framework proposed by Shulman (1986) introduced the notion of teacher content knowledge. Shulman's framework focused on the interconnections of teacher content knowledge and pedagogical content knowledge. In other words, how does what teachers know about content and pedagogy influence what their students learn. Successful technology integration for effective pedagogy requires developing technology, pedagogy, and content knowledge (TPACK) was later introduced. How can PTs develop positive attitudes for TPACK? Teacher preparation programs find themselves struggling with this view to prepare PTs (Niess, 2005). However, for PTs to become proficient with technology, teacher preparation programs must allow PTs opportunities to explore teaching with technology.

Describing TPACK, Niess (2005) wrote that it is the "integration of the development of knowledge of subject matter with the development of technology and of knowledge of teaching and learning" (p.510). TPACK involves the significance of three domains of knowledge: (PCK) pedagogical content knowledge, technological content knowledge (TCK), and technological pedagogical knowledge (TPK). As described by Mishra and Koehler (2006) PCK is knowledge of pedagogy that is relevant to the teaching of specific content. TCK is knowledge directly related to technology and content. TPK is knowledge related to technology and pedagogy.

Research Questions

Since PTs' beliefs and attitudes are critical for the development of TPACK, the research questions allowed for the PTs to explore growth in their view-points for the use of CAS in the teaching and learning of mathematics in the middle grades (Niess, 2006). Our research questions were:

(1) Are middle grades prospective teachers able to create a rich mathematical activity using CAS
technology?
(2) How does using CAS technology influence middle grades prospective teachers' philosophy for the teaching and
learning of mathematics?

It is our attempt to allow our PTs to explore the possible usage of CAS in the middle grades. In a content course designed for PTs of mathematics, we implemented rich mathematical activities using the TI-Nspire CAS and held workshops designed to increase their knowledge for the capacities of the Nspire CAS for the middle grades student.

Design of the Study

The study was conducted at Middle Tennessee State University, a public institution. The participants in the study were twenty (N = 20) middle grades PTs of mathematics. The PTs were registered for a middle grades mathematics education content course where connections to the middle school curriculum were addressed for such topics as number theory, trigonometry, and calculus. The study took place in Fall 2009 and the researcher was instructor of record. Of the 20 PTs, 85% were female and 15% were male. By ethnicity, 75% were Caucasian, 15% were African American, and 5% were both Vietnamese and Pakistani. In the course, the Taxes Instrument Nspire CAS was utilized. Each PT was issued a TI-Nspire CAS to use throughout the semester. The TI-Nspire CAS has capabilities similar to that of a graphing calculator, however with more advanced features as, symbolic manipulation, constructing geometric representations, and exploring multiple representations (i.e., algebraically and graphically) dynamically all on one screen.

The course made use of various assignments. Given that a major component of the course was content (e.g., number theory, trigonometry, and calculus) the PTs completed TI-Nspire CAS activities. Such activities were designed and/or modified by the instructor. For example, PTs were to explore the unit circle and the graph of the sine function while using the dynamic feature (all on one screen). The PTs also upload their completed instructor development and/or modified activities to the course web-site on Desire to Learn (D2L). These completed activities allowed the instructor to critique the PTs' content knowledge as well as their knowledge for the functionality of the TI-Nspire CAS. For instance, participates were to create a program that would export a quotient and remainder after a positive integer for the dividend and divisor was imputed. These instructor development and/or modified activities also allowed the PTs to strengthen their knowledge for the capabilities for using the TI-Nspire CAS with middle grades students. PTs also completed weekly journals. The weekly journals served two purposes: 1, allowed the PTs to reflect about their technology views, beliefs, and attitudes; and 2, communicate about his/her conceptual understanding of the mathematics in the instructor development and/or modified activities.

At the beginning and end of the course, the PTs completed a pre- and post- questionnaire respectively. The pre-questionnaire also elicited demographic information. The pre- and post-questionnaires were identical, however, the demographic information was omitted from the post-questionnaire. The demographic information included such items as: gender, classification in college (e.g., junior, senior, etc.), mathematics classes taken at the secondary level, and any mathematical technology used. As for the reminder of the pre- and post questionnaire, PTs answered such questions as:

1. Should mathematics teachers use technology (mathematics specific)? Explain.

2. Thus far, is technology a part of your philosophy for the teaching and learning of mathematics? Explain.

3. Do you think CAS technology should be used in the middle grades (6-8)? Explain.

Last, the PTs were to create an activity that could be used by middle grades students. The PTs worked in groups of two. Each group then selected a middle grades state standard. This standard would be the rationale for each group's activity. Each group was required to create and/or modify an activity illustrating their state standard using the TI-Nspire CAS. More specifically, the the activity was to use the functionality that distinguishes the CAS from a graphing calculator – the symbolic manipulation. The PTs also presented their activity near the end of the course. At this time the instructor recorded notes and asked various questions to help drive the idea of CAS. Such questions asked were: how does your activity use the functionality of CAS? How does using CAS allow middle grades students to explore the state standard?

Data Analysis

In order to answer both research questions we determined that descriptive statistics and qualitative data would be most appropriate. The data analysis was ongoing throughout the study, which in turn directed the focus and outlined the research as it progressed (Erickson, 1986). The research team decided to approach data collection and analysis as an iterative process. We decided that the data collected would be triangulated with the research questions.

The demographic data from the pre-questionnaire were analyzed. This examination consisted of using descriptive statistics for the following demographic information: gender, classification in college (e.g., junior, senior, etc.), mathematics courses taken at the secondary level, and any technology tool used. Percentages and charts of the descriptive data were computed and created.

TPACK framework was used to further guide the analysis of the qualitative data. The qualitative data used in this analysis were the completed instructor developed and/or modified activities, weekly journals, developed activities using a state standard, and observer notes during presentation. The analysis of the qualitative data contained three parts. First, in order to categorize the qualitative data, the research team constructed a matrix identifying the research questions and categorized the data accordingly. By doing so, the research team was able to investigate a specific research question and able to conduct several readings without interrelated themes (Creswell, 2003; Patton, 2002; Wiersma, 2000). Next, the research team engaged in several more readings of the qualitative data in order to code the data based on the three domains of TPACK (PCK, TCK, TPK). After multiple readings, the research team was able to create a list of introductory codes, in each domain of TPACK, based on the themes presented in the qualitative data according to the research questions. Third, the research team then continued with another set of readings of the qualitative data without the introductory codes. This was done to create accuracy in the codes that would be used in the final analysis. After further scrutiny of the introductory codes and the later codes, the research team was able to identify specific codes that would be used to analyze the qualitative data (Patton, 2002). Last, the research team then took the various codes from the qualitative data according to the research questions and categorized them into codes that were emergent throughout the qualitative data.

Results

The results section begins by providing a description of the demographic data. Twenty pre-questionnaires were issued, 85% were returned. This is followed by two major themes presented from the data. First, PTs change position to indentifying CAS not just for examining secondary or collegiate mathematics but as a technological tool to allow middle grades students' opportunities to explore mathematical phenomenon. Second, PTs view that any technological tool can be used to help students explore mathematics if used appropriately.

The results presented in the demographic data provided the research team with PTs'' secondary mathematics classes and familiarity with various mathematical technological tools. In Figure I, the data represented a wide range of mathematics classes at the secondary level. Such classes as geometry (100%), Algebra II (100%), Algebra I (74%), pre-calculus (65%), trigonometry (18%), advanced algebra/trigonometry (18%), and pre-algebra (6%). We also noticed in the data that PTs took advantage of advance placement classes; such classes as advanced placement calculus AB (35%) and advanced placement statistics (12%). Figure II illustrates the PTs experience with mathematical technological tools, wide ranges were reported. Such tools as graphing calculator (88%), fraction/scientific calculator (76%), spreadsheets (71%), access information on the web (59%), presentation devices (53%), multimedia software (35%), data collectors (35%), dynamic geometry software (24%), and topic-specific math software (12%). The data also reported that 29% of the PTs had previous experience using a computer algebra systems.

Research Question 1: PTs Activity Development Using CAS

In the development of TPACK, for this research, PTs using CAS to create a rich mathematical activity for middle grades students was reflected in the qualitative data. For this development to occur, PTs continued progress toward PCK, TCK, and TPK is critical. Most PTs'' demonstrated effective use of PCK, TCK, and TPK. Figure III illustrates evidence of TCK as identified in one group's activity. This data indicates that these PTs allow the middle grades student to explore the functionality of CAS, while providing some guidance to further develop student's mathematical understanding. Although, there exist a feature on the TI-Nspire CAS that will allow the user to simply input *solve (7a-17=60,a)* and the output of *a=11* will result. Instead, middle grades students must perform or show each step to solve the equation. This notion displays TCK; using the technology to help explore the content.

The same activity also displays PCK. In Figure IV, a middle grades student must first determine to add, subtract, multiply, or divide. The activity then instructs the middle grades student to multiply, though, not indicating that multiplication is the operation that should be completed first. Then, the student is to solve the the same equation using subtraction as the first operation. Still supporting the notation that this activity provides evidence of PCK, the middle grades student is exploring solving the equation. While completing this portion of the activity, the student is to make conjectures about solving two-step equations; then, test those conjectures. Typically PTs create lesson plans and/or activities that do not require students to make conjectures.

Last, the activity also demonstrates evidence of TPK. Figure V demonstrates that the activity instructs the middle grades student to perform two different operations for the same equation. This allows the students to explore a different operation being used and determine a rational for the solution generated by the TI-Nspire CAS. The activity continues by instructing the student to graph both sides of the equation not informing the student that the intersection is the desired solution. The student comes to this conclusion after graphing each step of their work. Once done, there is a common x value of importance (while the y value changes).

Similar to other PTs, this group demonstrates evidence of TPACK. Middle grades students would use the TI-Nspire CAS to explore mathematical phenomenon. Instead of being told specific rules (without meaning) and doing several of the same type exercises, students create conjectures and test them using CAS.

Research Question 2: PTs' Changed Philosophy

Of the twenty PTs, seventeen demonstrated growth leading to TPACK. These seventeen PTs previously valued using technology to aid in the teaching and learning of mathematics. Many expressed their willingness to use the TI-Nspire CAS to allow middle grades students' opportunities to explore mathematics; though, there were three PTs who showed resistance. One PT was adamant that the TI-Nspire CAS would not be supported by school administration nor would a school district fund such a technology for the middle grades. These negative connotations, as written by the PT, come directly from the PT's mentor teacher. This PT wrote:

> Yes, I plan to use technology in my classroom as much as I can because I feel students will connect with this approach to learning. This is a new era and students and teachers need to understand the technology that is out there that can help us understand the concepts in mathematics completely. I am concerned about the amount of technology that is available in the school systems. When introduced to the CAS, I was very impressed but wondered if the school system I am interested in obtaining a job would invest in this technology. I then went to my mentor teacher that I have in this school system and asked about the technology that is available to them. She then explained that this tool [CAS] was not available in that school but was used by one teacher in another school in the same county. She not sure if this was the CAS but when she explained the features on the tool, it is very similar if not the same.

Even though the mentor teacher expressed their opinions, this PT is highly encouraged to use technology in the teaching and learning of mathematics.

The second PT was confident that technology should be used in the middle grades, however, wrote that CAS is too advanced. This PT's initial teaching philosophy regarding technology was:

> Technology is a part of my philosophy for the teaching and learning of mathematics. Technology is everywhere and is growing more vital every day. Students should be exposed to technology and it can effectively be built into mathematics lessons.

In the pre-questionnaire, this PT's comment about using CAS in the middle grades was:

> No, I do not think the CAS technology should be used in the middle grades. Again I believe that this calculator is much too advanced and at these grades the students are learning the basics and some students will get confused with this advanced of a calculator.

Throughout the course, it was not the intention of the instructor to force the PTs to use CAS with middle grades students; rather, acquaint the PTs to another form of technology that could be used with middle grades students. Later in the course, this PT wrote in the post-questionnaire:

> I am slowly starting to see the value of the N-spire and do believe that when I start teaching, I probably will try to get a class set for my students to use. I still believe that the N-spire is very expensive; however, it does have its benefits. Regardless of whether I have a class set of N-spires or not, I will definitely use the N-spire teacher edition software on the projector for notes and examples in my lessons.

The instructor of the course used various components of the TI-Nspire CAS so that PTs were able to explore the capabilities. Some of the features identified by this PT were not related to the CAS. This PT mentioned receiving class notes on the Nspire and the dynamic representations. All of which can be accomplished on the TI-Nspire non-CAS.

The last PT who disliked the TI-Nspire CAS for middle grades students, wrote in the pre-questionnaire that technology was not considered in their philosophy for the teaching and learning of mathematics:

> Well, I haven't put a lot of thought into my teaching philosophy yet. I have written out my teaching philosophy a couple of times, but never once was technology mentioned. I should probably consider technology to be used when I teach. Technology is a great tool for teaching as long as the teacher knows how to handle it. You do not want to exploit it too much, because it might promote laziness and dependable characteristics.

This PT views technology as a hindrance to student learning. According to this PT, students become dependent on the tool rather than *knowing* the mathematics. Midway through the course, this PT's view for using technology evolves, although, their view for using technology is reported from a learner of mathematics rather than a teacher of mathematics:

> This week I feel like the Nspire is kind of beneficial for me. There are so many different functions in this calculator. There are so many different functions just like the Spreadsheet. I definitely appreciate the visual presentation of this calculator. It blew my mind when I saw the transformation of different kinds of graph. For my understanding, students could make polygons and shape using the Nspire. So some individuals will find this really interesting, and somewhat time consuming. It definitely takes me some time to get use to this calculator, so I wonder how long it will take for students to get use to the

> Nspire. Younger kids are more susceptible to different types of technology than adults. I understand that but I'm still deciding if this is a good tool for me.

Additionally, this PT does make reference to their future students yet continues to write from the perspective of what works for the PT's mathematical understanding. Far too often, PTs view mathematics teaching to benefit their own understanding instead of viewing the learning from the perspective of their future students. Near the end of the course, this PT's attitude moved from having not thought about technology to allowing middle grades students' opportunities to use CAS to explore mathematical phenomenon:

> Yes, CAS is very helpful for them [students] to see the way to solve for x and y and also see the graphs. I think it is very useful for the students to see the equations and then see the visual graphs.

In the end, the three PTs made tremendous gains in their beliefs for using technology in the teaching and learning of mathematics. All qualitative data were consistent, although the largest gains were made with the three PTs described above.

Conclusion

This research sought to investigate middle grades PTs using the TI-Nspire CAS to create rich mathematical activities and examine how using CAS influence PTs' philosophy for teaching with technology. Through this research, two major themes are present, 1) PTs change position to indentify CAS as a technological too to enhance middle grades students' mathematical understanding, 2) PTs view that any mathematical technological tool has the potential for middle grades students to explore mathematics.

The study revealed that PTs were able to design a rich mathematical activity using CAS technology to help middle grades students examine mathematics. These activities support the learning of mathematics without losing rigor. The use of CAS technology does not make mathematics less rigorous, rather promoting developer insight and fostering mathematical curiosity (Cuoco & Manes, 2001, Herget, Heugl, Kutzler, & Lehmann, 2001, Pierce & Stacey, 2001, 2004). Heid and Edwards (2001) report CAS in the classroom can produce higher order thinking for students.

> A CAS-present curriculum can result in: more realistic problems, deeper exploration of mathematical concepts, increased opportunities to develop connections among mathematical ideas, a wider range of examples, more abstraction, a more complete set of examples and non-

> examples in a shorter period of time, and new ways to understand traditional procedures (p.132).

Similar to Bossé and Nandakumar (2004) many of the PTs in the current research study were consistent that "CAS should be employed when doing so would be pedagogically and epistemologically sound and when it would enhance student learning" (p.298). It was our attempt to promote mathematical understanding for all students sing CAS.

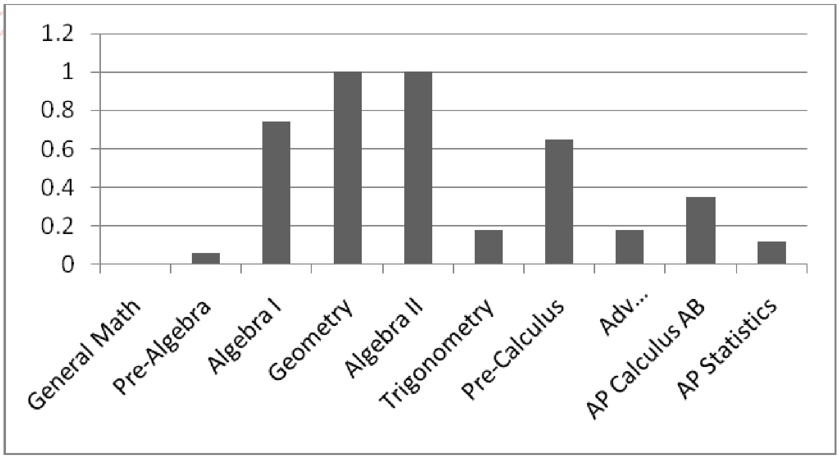

Figure I. Secondary mathematics classes

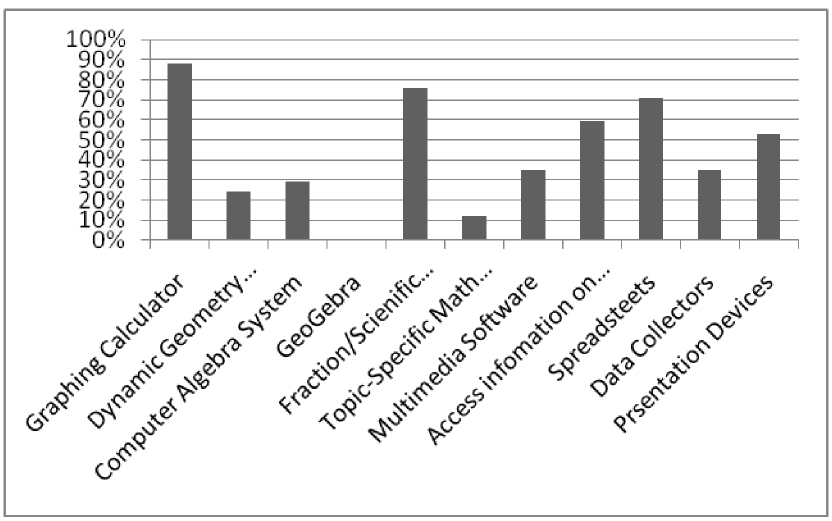

Figure II. Mathematical technological tool experience

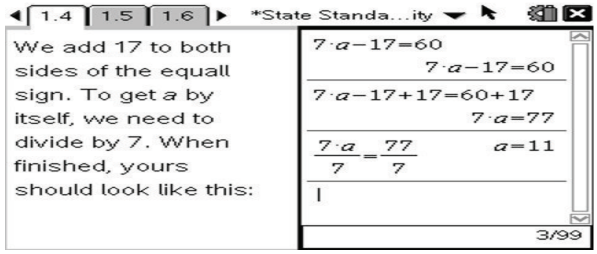

Figure III. TI-Nspire CAS screen capture: activity demonstrating TCK

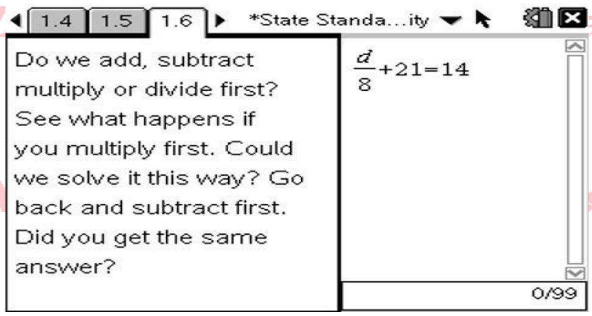

Figure IV. TI-Nspire CAS screen capture: activity demonstrating PCK

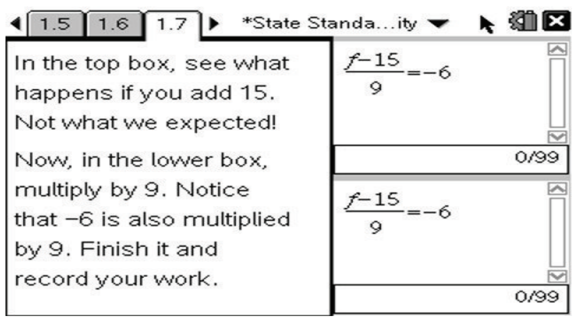

Figure V. TI-Nspire CAS screen capture: activity demonstrating TPK

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