Trigonometry Students' *Knowing* When To Use Hand-Held CAS Technology To Make Sense of Mathematics

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Introduction

Research has shown that hand-held Computer Algebra Systems (CAS) is an effective and valuable tool to enrich and promote the instruction of mathematics (Bossé & Nandakumar, 2004; Day, 1993; Vlachos & Kehagias, 2000). CAS performs manipulations accurately and quickly. Students can obtain both exact and approximate results without worrying about tedious steps and errors. Researchers argue that teachers can use the efficiency gained from using CAS to focus on conceptual development, problem solving, and investigations with realistic problems (Cuoco & Manes, 2001; Hillel, 1993; Herget, Heugl, Kutzler, & Lehmann, 2001; Pierce & Stacey, 2001, 2004). Furthermore, weak students greatly benefit from the accuracy and immediacy of CAS. Instead of getting stuck in routine procedures, students experience more complex task such as making connections between an algebraic expression and a graph (Pierce, 2005).

Why Technology?

Garofalo, Drier, Timmerman, and Shockey (2000) suggests that the effective use of technology should entail the following components: an introduction of the context, address the worthwhile mathematics with appropriate pedagogy, take advantage of the technology, connect mathematics topics, and incorporate multiple representations. An introduction of the context should be illustrated in the context of meaningful content-based activities. Teachers should teach a set of technology or software-based skills and then attempt to find mathematical topics for which they might be useful in the instruction of mathematical procedures. Addressing the worthwhile mathematics with appropriate pedagogy, addresses the procedures, strategies, and should reflect the nature and spirit of mathematics. The activities should support sound mathematical curricular goals and objects; they should merely not be developed because technology makes them possible. Garofalo et al (2000) discussed taking advantage of the technology solicits that activities should take advantage of the capabilities of technology and should extend beyond or significantly enhance what could be done without technology. Technology should enable the students to explore topics in more depth (e.g., interconnect mathematics topics, write programs, devise multiple proofs and solutions) and in more interactive ways (e.g., simulations, data collection of probes). Technology also makes more accessible the study of mathematics topics that were previously impractical, such as recursion and regression, by removing computational constraints.

In order to connect mathematics topics, Garofalo et al (2000) further state that technology-augmented activities should facilitate mathematical connections in two ways: (a) interconnect mathematics topics and (b) connect mathematics to real-world phenomena. Appropriate use of technology can facilitate such connections by providing ready access to real data information, by making the inclusion of mathematics topics useful for applications more practical (e.g., regression and recursion), and by making it easier for teachers and students to bring together multiple representations of mathematics. The last key aspect, according to Garofalo et al (2000), is the incorporation of multiple representations. Technology activities should incorporate multiple representations of mathematical topics. The researchers note that many students who have difficulty connecting the verbal, graphical, numerical, and algebraic representations of mathematical functions can have those concerns addressed by the appropriate use of technology. The appropriate can be effective in helping students make connections (e.g., connecting tabulated data to graphs and curves of best fit, generating sequences and series numerically, algebraically, and geometrically).

The influence of integrating technology into the mathematics classroom (Harvey, Waits, and DeMana, 1995) stands out as the greatest revolutionary influence on mathematics education. The impact could shift the focus of school mathematics from a dualistic mission to a singular focus on a significant common core of mathematics for all students, the instruction of mathematics is shifted from an authoritarian model based on “transmission of knowledge” to a student-centered practice featuring “simulation of learning”, the instruction of mathematics is shifted from preoccupation with inculcating routine skills to developing broad-based mathematical power, the instruction of mathematics is shifted from emphasis on tools for future coursed to a greater emphasis on topics that are relevant to students’ present and future needs, and the instruction of mathematics is shifting from primary emphasis on paper-and-pencil calculations to full use of calculators and computer.

Hand-Held CAS Technology

Asp and McCrae (2000) reported hand-held CAS allows teachers and students alike to explore algebraic functions in ways that was once unavailable at the secondary levels. The device permits teachers to introduce and instruct mathematics in many different ways. The device also gives students the opportunity to solve, factor, differentiate, and integrate Algebra functions; thus not making them totally dependent upon their pencil and paper Algebraic skills.

Ball and Leigh-Lancaster (2001) state hand-held CAS has proven to be a powerful learning tool that allow students to move between numeric, graphical, and symbolic representations of a problem. They also note that hand-held CAS also allows students to observe patterns and explore concepts that promote understanding of mathematical procedures. According to Pierce and

Stacey (2002), effective use and value of hand-held CAS depend upon how effective the tool is utilized in the classroom. Employing hand-held CAS requires that the student becomes familiar with both the hardware and software that is associated with this technology. This presents the students with two learning outcomes: (a) learning the technology; and (b) learning the mathematics. To obtain the student learning objectives, the teacher must ensure that students learn how to operate the device with a minimum of difficulty and have a positive attitude toward the operation of hand-held CAS.

Similar to Garofalo et al (2000), Guin and Trouche (1999) report the integration of hand-held CAS technology into classroom instruction gives students the opportunity to explore many mathematical concepts graphically, algebraically, and symbolically. The emphasis on memorization and rigorous algebraic manipulation skills would be replaced with realistic and real-life problems. The students would be required to answer questions that compel a more in-depth analysis and comprehension of the matheamtics. However, in order to achieve this level of critical thinking, the teacher must provide innovative and creative ways of designing questions due to the technology.

Connors and Snook (2001) also reported that the usage of hand-held CAS technology empowers students to understand concepts and solve problems. They cite an improvement in student improvement in conceptual understanding and maintenance of procedural skills, and an improvement in problem solving. Other investigation conducted concluded an increase in the percentage of students who enjoyed the calculus course. It is carefully noted that an avoidance of letting a powerful technology tool become a crutch can lead to a poor understanding of the concepts. Instructors and students need to maintain a delicate balance of technology use in their quest for excellence. Connors and Snook (2002) concluded that students learn how to use hand-held CAS technology operations to manipulate and solve systems of equations. Linear Algebra capabilities such as computing eigenvalues and eigenvectors assist students in finding analytic solutions for systems of equations and analyzing a system's long-term behaviors. Hand-held CAS symbolic, graphing, and table features assist in developing students' understanding of the calculus concepts of limits, derivatives, optimization, points of inflection, and definite/indefinite integrals. Students can write user-defined functions and programs to develop the concept of Riemann sums and areas under a curve.

Garner and Pierce (2005) conveyed that students who use hand-held CAS all the time will not necessarily be faster in their mathematical processes. Entering expressions correctly and dealing with unexpected results can take both time and disrupt mathematical thinking. Equally the student who solves every problem by pencil and paper will also not be the most efficient. Working with hand-held CAS can allow students to perform challenging calculations or manipulations both quickly and correctly. Garner and Pierce (2005) encourage students to be discriminating in their functional use of hand-held CAS; that is, they choose to use hand-held CAS for lengthy computations, but choose pencil and paper for quick solutions and /or straightforward calculations. Such behavior requires a disposition to plan ahead, considering the nature of the task, in addition to mathematical and technical skills. Students should be ready to respond to the availability of hand-held CAS, and the expectation to use this technology in different situations. Hand-held CAS provides students with an additional tool, which, when used strategically, that is in a well planned and discriminating manner, may assist students to explore mathematics and tackle problems that they may previously have found daunting. It is suggested that the teacher takes advantage of hand-held CAS and to both model effective use of hand-held CAS and support their students in making sound decisions about when and how to use this new tool.

Purpose

Others have researched the influence of CAS on student learning of mathematics (Bossé & Nandakumar, 2004; Day, 1993; Johnson, 2010; Vlachos & Kehagias, 2000), while some have provided theoretical perspectives for the use of hand-held CAS (Heck, 2001; Zand & Crowe, 2001); the current research aims to explore students' ability to *know* when to use hand-held CAS technology to make sense of mathematics. By *know,* we mean when students have a desire to explore a topic in mathematics though the student may have an understanding or trying to grasp that concept. We also examined the students' position about the use of hand-held CAS technology to make sense of mathematics.

Method

The research consisted of descriptive statistics and qualitative data structures for one section of Plane Trigonometry at Middle Tennessee State University, a public institution where lead researcher was instructor. Plane Trigonometry at the research site is designed for non-mathematics majors. Typical majors in the course were construction management, criminal justice administration, and medical diagnostic sonography. Students who take Plane Trigonometry also take a prior mathematics course such as College Algebra, Mathematics for General Studies, or College Mathematics for Managerial, Social, and Life Sciences. Typical topics in Mathematics for General Studies are logic, sets, algebraic reasoning, probability, statistics, and consumer mathematics. College Mathematics for Managerial, Social, and Life Sciences consist of topics as solving systems of linear equations, Leontief models, linear programming, mathematics of finance, set theory and probability theory. Since no mathematics beyond Plane Trigonometry is required for such majors, Plane Trigonometry is commonly the last college mathematics course these students will take. However, some students will opt to pursue Pre-Calculus and then possibly Calculus I.

The Plane Trigonometry course enrolled twenty-two students. Of the twenty-two students only sixteen returned the pre- and post-questionnaires. The research team determined that the unit of analysis would be the sixteen students. The gender of the sixteen students was both eight for female and male. Each student enrolled in the lead researcher's section of Plane Trigonometry received a Texas Instrument (TI) Nspire CAS. The TI-Nspire CAS is a hand-held system with more advanced features than a graphing calculator. As noted by Johnson (2010), the TI-Nspire CAS has "advanced features as, symbolic manipulation, constructing geometric representations, and exploring multiple representations (i.e., algebraically and graphically) dynamically all on one screen" (p. 44).

Data

As apart of the course, students were required to complete various assignments. These assignments were also used as data for current research study (not including quizzes or tests). Students completed pre- and post-questionnaires, technology use questionnaire, writing prompts (n=4), and weekly journals (n=14). First, Figure 1 illustrates the twenty-two likert scale questions on the pre-questionnaire (agree, neutral, or disagree). The research team selected an existing questionnaire where triangulation with the qualitative data would be possible. For this reason, the research team determined that the questionnaire developed by Stewart, Thomas, and Hannah (2005) contained relevant likert scale questions that would help answer the research question. However, Stewart et al (2005) did not investigate students' ability to *know* when to use technology, rather, the instrumentation of CAS by college students. Moreover, their questionnaire does allow students' to provide his/her attitudes for using technology to learn mathematics; which provided the current research study an opportunity to examine the Plane Trigonometry students' position. Example likert scale questions were "technology does not improve my understanding of mathematics" and "I often check my answers using technology." The pre-questionnaire also included four open-ended questions. Such questions were "how do you decide when to use technology to help you learn mathematics?" and "what do you like about using technology to help you learn mathematics?" Next, the post-questionnaire consisted of the same twenty-two likert scale questions and six open-ended questions. The research team determined that the open-ended questions should reflect the students' experience using the TI-Nspire CAS. Such questions were "how did you decide when to use the TI-Nspire CAS to help you learn mathematics?" and "did the TI-Nspire CAS enhance your knowledge of the content in the course? If so, explain. If not, why not?" The open-ended questions in both the pre- and post- questionnaires will allow the research team to study the Plane Trigonometry students' position about using the TI-Nspire CAS and when they decide to use the technology to make sense of mathematics.

A third data source was the technology use questionnaire. This questionnaire was divided into four parts. The intent of the technology use questionnaire was to inform the research team of the students' prior experience with technology to learn mathematics. The technology use questionnaire also allowed the research team to identify the types of technology used to learn mathematics. Part one required the students to identify any technology used to help learn mathematics in three different grand bands – PreK-5, middle grades, and secondary levels. This part of the questionnaire was a prescribed checklist where students would identify any technology used at the various grade bands. The prescribed checklist was the same for all three grade bands. Such technologies listed were four-function calculator, dynamic geometry software, and computer assisted programs. Next, if no technology was listed for any grade band the students were to indicate why technology was not used. Again, this was a checklist where students would simply check a prescribe list of responses. Such responses were "school did not provide the resources" and "teacher did not promote." The third part of the technology use questionnaire required students to determine how often technology was used at each grade band. Similar to the preceding parts, students would check the most appropriate time frame. Such time frames were "1-2 times a week" and "everyday." The last part was two open-ended questions; one question was "do you think technology has the potential to enhance your knowledge of the content in this course? Explain." The technology use questionnaire will allow the research team to identify the Plane Trigonometry students' prior experience with using various types of technology to learn mathematics.

Fourth was the writing prompts (n=4). The writing prompts allowed the students to express their understanding of Plane Trigonometry in written form. All writing prompts were no more than six questions. For example, one writing prompt question required the students to write a poem or short story using a list of prearranged Plane Trigonometry terms. Another asked the students to discuss the contributions to Plane Trigonometry for Ancient Babylonians, Ancient Greeks and Medieval Hindus. More specific to technology, Figure 2 illustrates question three of writing prompt I, where students were required to explore the relationships for the following $\frac{w}{e}, \frac{k}{e}, \frac{w}{k}, \sin(\theta), \cos(\theta),$ and $\tan(\theta)$ based on the triangle provided. Students were then asked to create conjectures that would be tested during class instruction. The writing prompt questions motivated by technology, encouraged the students to explore the power of the hand-held CAS to make sense of mathematics.

The last data source was the weekly journals (n=15). The weekly journals were stored electronically on the class website on Desire2Learn (D2L). The purpose for the weekly journals was to allow the research team to view a weekly snap-shot for the Plane Trigonometry students' attitudes for using the Hand-held CAS to make sense of mathematics, their knowledge of Plane Trigonometry, and their ability to use or not use the Hand-held CAS effectively.

Analysis of Data

The research team determined that descriptive statistics and qualitative data would be most suitable to answer the research question. The research team decided that the analysis of data would be an ongoing process. The data collected would then be triangulated with the research question also noting any emergent themes.

The technology use questionnaire and both the pre- and post- questionnaires were analyzed. This analysis involved using descriptive statistics for both the likert scale data in the pre- and post- questionnaires and technology use questionnaire, each was completed separately. Percentages of the descriptive data were calculated and charts were created.

The qualitative data used in this analysis were the pre- and post-questionnaire, technology use questionnaire, writing prompts (n=4), and weekly journals (n=15). As previously stated, the pre- and post-questionnaire and the technology use questionnaire contained open-ended questions that were used in this analysis. The analysis of the qualitative data contained three parts. First, the research team conducted several readings of the qualitative data. By doing so, we were able to identify any emergent themes (Creswell, 2003; Patton, 2002; Wiersma, 2000). These emergent themes helped to create the introductory codes. Next, after several more readings of the qualitative data without the emergent themes and introductory codes, created in the initial readings; we then identified another set of emergent themes in an attempt to create additional introductory codes. We determined that phase two of the analysis was critical such that the accuracy of these codes would be used in the final analysis. After further scrutiny of the introductory codes and the later codes, we were able to identify final codes that would be used to analyze the qualitative data. Third, we continued with another set of readings of the qualitative data using the final codes that were scrutinized and categorized these final codes into the final emergent themes.

Results and Discussion

In the following section, the data analysis will be presented in three parts. First, Plane Trigonometry students' prior experience with various technologies to make sense of mathematics. Followed by, students position about using technology to make sense of mathematics. Last, students' ability to *know* when to use hand-held CAS to make sense of mathematics.

Students Prior Experience with Using Technology to Make Sense of Mathematics

The results of the technology use questionnaire illustrated approximately 56% of the students utilized some form of technology to make sense of mathematics at the elementary levels; while approximately 44% reported that technology was not used. The data further showed, as demonstrated in Figure 3, 56% of the students who indicated that they used technology to make sense of mathematics; such technologies used were the internet (31.25%), computer software programs (25%), and email (18.75). After additional analysis of the 56% of the students who reported technology was used to make sense of mathematics, 66.7% indicated technology was used 1 or 2 times a week, while 22.2% indicated 3 or 4 times a week. 11.1% did not report how often technology was used and zero percent reported everyday usage. Further analysis demonstrated that of the 44% who responded technology was not used to make sense of mathematics, 14.28% indicated school did not provide resources and 42.86% teacher did not promote. 28.58% of the students responded to both school did not provide resources and teacher did not promote. 14.28% did not report a reason why technology was not used. An alarming data was zero percent of the students indicated that manipulatives were used to make sense of mathematics at the elementary levels. This is dissimilar to other research studies, where manipulatives were highly used to make sense of mathematics at the elementary levels (Burns, 2005).

As for the Plane Trigonometry students' experience with technology to make sense of mathematics at the middle grades levels, approximately 94% of the students utilized various forms of technology to make sense of mathematics at the middle grades levels; while approximately 6% reported that technology was not used. Figure 4 illustrates of the 94% who reported that technology was used to make sense of mathematics such technologies used were four-function calculator (81.25%), Internet (43.75%), scientific calculator (37.5%), graphing calculator (37.5%), cellular phones (25%), TI-Nspire (18.75%), clickers (6.25%), CAS technology (6.25%), and IPODs (6.25%). Additional analysis indicated, 94% of the students who reported technology was used to make sense of mathematics indicated the frequency of technology was 60% 3 or 4 times a week, 33.3% 1 or 2 times a week, and 6.7% everyday. Further analysis showed that of the 6% who responded technology was not used to make sense of mathematics, 6% did not report.

Last is the students' secondary experience with technology to make sense of mathematics. The data indicated that approximately 100% of the students utilized some type of technology to make sense of mathematics at the secondary levels. Such technologies used were scientific calculator (81.25%), graphing calculator (81.25%), four-function calculator (62.5%), Internet (62.5%), cellular phones (56.25%), email (31.25%), computer software programs (18.75), IPODs (18.75%), and Web 2.0 (12.5%), as illustrated in Figure 5. After auxiliary analysis, 100% of the students who reported technology was used to make sense of mathematics indicated the usage of technology was 62.5% everyday, 31.25% 3 or 4 times a week, and 6.25% did not report.

Students' Position For Using Technology to Make Sense of Mathematics

According to the data, 100% of the Plane Trigonometry students' position for using technology to make sense of mathematics was positive, though one student's view changed to positive over time. After additional triangulation, one emergent theme was technology is useful to make sense of challenging problems. This theme was consistent through all the data sources. There were two students Miguel and Nathan who projected this theme immensely. For instance, on the pre-questionnaire, Miguel reported:

> My main problem in math is making a small mistake that tends to cause me to totally miss the problem. When I have a calculator I will usually check within a problem to see if I have correctly solved part of a problem. I struggled in algebra in high school, partly due to my teachers, and we did not use

> calculators. It was difficult for me to understand the concepts in high school algebra. Using technology helps me to focus on the concepts and formulas in math, as opposed to concentrating on getting every single number correctly.

It is clear that Miguel has the view that technology allows him to make sense of mathematics, rather than focusing on numeric calculations. Nathan also engaged in the idea that technology used to make sense of mathematics was beneficial. Nathan wrote in journal 2:

> The Nspire CAS continues to be a wonderful tool in helping me not only to find the correct answer, but to find a way to work the problem while learning several steps in the process. I feel that I still have much to learn as far as the Nspire CAS's functions and capabilities are concerned.

As for Miguel's progression in the course with using the hand-held CAS, his view has not changed, though, enhanced. For the post-questionnaire, Miguel wrote:

> I believe that the TI-Nspire CAS helped me greatly in understanding certain trigonometric functions. The ability to move points on a triangle or circle and see corresponding values in a function helped me a lot. It is also neat having the notes on a calculator.

Nathan also conveyed a positive view for using technology to make sense of mathematics over time. Identified on the technology use questionnaire (administered at the beginning of course), he reported, "Technology has the potential to make this course easier. To an extent, it will enhance knowledge; in the resources such as the internet provide examples and online tutors." However, Nathan's perspective for such technologies as calculators and/or hand-held devices seems to be less positive. On the pre-questionnaire Nathan wrote:

> We are taught the majority of our math on a calculator and ultimately have no idea how to apply these equations and theories. For example, with a calculator I can find an answer to most logarithms. However, if left to do this by hand I would be completely clueless. I like the convenience of technology, but I feel that we lose some of the effect of learning.

It was observed that Nathan views technologies such as a calculator and/or hand-held device to be a hindrance to make sense of mathematics, thought; he understands the ease for using the calculator.

As the course progressed, Nathan's position for calculators and/or hand-held devices changed when he was able to make sense of mathematics using hand-held CAS technology. Nathan has a positive view for technology to make sense of mathematics; however, he struggles when the use of a calculator and/or hand-held device should be used. Nathan's new perspective for calculators and/or hand-held devices was observed in the post-questionnaire. He wrote "Yes, I felt that the technology [Nspire CAS] allowed us to delve farther into the material. We could forgo the more basic skills and focus on learning that which is more difficult." In Nathan's final perspective, he used the term *technology* when referring to the Nspire CAS. The course never made use of the internet and/or online tutors, examples for technology mentioned in his technology use questionnaire, to make sense of mathematics. The only technology used to make sense of mathematics was a hand-held CAS.

Students' Ability to Know When to Use Hand-Held CAS to Make Sense of Mathematics.

Next, we explore when students' use hand-held CAS to make sense of mathematics. Figure 6 demonstrates the results of the pre-questionnaire likert scale questions. Question thirteen asked if students found it difficult to decide when to use technology when doing mathematics problems. The data revealed that 50.00% disagreed, 43.75% neutral, and 6.25% agreed. Further analysis of the qualitative data on the pre-questionnaire helped to identify cases when the students used technology to make sense of mathematics. Question one of part II of the pre-questionnaire, asked students how did they know when to use technology to make sense of mathematics. The data indicated 37.50% when a mathematics problem seemed difficult to learn, 37.50% when basic mathematics was required which made the completion of work faster or quicker, and 25% to verify pencil and paper work and solution. On the pre-questionnaire, Miguel wrote:

> I like to use calculators to solve the complicated arithmetic in math and I use the calculator to check my answers often. I still choose to write the problem down and see how I got the solution, as opposed to just typing in numbers and getting an answer. I hardly ever attempt to learn new material through the internet, because it is difficult for me to learn without someone showing me or telling me how to do something. A calculator is the only technology that has been helpful for me in mathematics.

Miguel's position for using technology was consistent with his view for when he used technology to make sense of mathematics. Other students agreed with this perspective too. He mentioned that the internet was not a technological tool he preferred; given that he needed someone to explain the steps to solve mathematical problems. The position that he desired to understand how he arrived to a solution, rather than just typing numbers into the technology to generate an answer or solution, was also consistent with other students.

Nathan's pre-questionnaire revealed a similar perspective. Nathan wrote "Due to being trained to do many of the intermediate and advanced mathematics via technology, for me I seem to resort to technology whenever a problem goes beyond the most basic skills." He too indicated that he used technology to do more rigorous mathematics. The perspective that the technology was useful to achieve lower level mathematics was consistent through all data sources.

Analysis of the post-questionnaire indicated similar results as the pre-questionnaire. Question thirteen revealed that 50.00% disagreed, 43.75% neutral, and 6.25% agreed. Additional analysis of the qualitative data question one of part II indicated 56.25% when a mathematics problem seemed difficult to learn and 43.75% when basic mathematics was required which made the completion of work faster or quicker. From the post-questionnaire, students who used the hand-held CAS to solve problems that seemed difficult increased by 18.75%. For instance, question three of writing prompt III, illustrated how students utilized the hand-held CAS to make sense of mathematics. Question three required the students to:

> Draw a circle with a diameter. Next, measure the length of the diameter. Mark the center of the circle point M. Then choose 3 points A, B, and C on the circle so that when you draw a triangle with these points as vertices, M is inside the triangle. Measure the angles and side lengths. Then, find $\frac{a}{\sin(A)}$, $\frac{b}{\sin(B)}$, and $\frac{c}{\sin(C)}$. Compare these ratios to the diameter and radius of the circle. What do you notice? What relationships are true? <u>Use your hand-held CAS!</u>

Figure 7 illustrates Miguel's response to the above writing prompt question. The sketch provided was correct and all essential information was found. He indicated a diameter 16.88 cm and radius 8.44 cm and calculated all required angle measurements and ratios; though, he failed to determine a relationship comparing the law of sines to the diameter and radius. Miguel reported that "AB = BC = CA; all sides of the triangle still abide by the law of sines." The solution Miguel recorded is correct, however, lacks a decision about any conjectures regarding the diameter and radius. For this writing prompt question, students were obligated to use their hand-held CAS to demonstrate how hand-held technology can be used to make sense of mathematics.

Since no two students should have the same size circle, Nathans solution varied. Nathan's solution to the same question is demonstrated in Figure 8. Nathan successfully calculated the needed ratios, however, omitted the diameter and radius for the circle created. In his decision, he wrote "The sides of the triangle & the diameter are practically equal. To me, this proves that triangles are a more practical method to finding the diameter of a circle." At first glance, it could appear that Nathan has an understanding for the concept, however, he calculated the area of the circle 114.556 cm^2 and $\sqrt{114.556}$ = 10.7030 cm. We could not determine if Nathan used these two values to verify his solution. The course made use of various assignments that encouraged the students to engage with hand-held technology to make sense of mathematics. After being permitted to to *play* with technology, it was our attempt to allow the students to make conjectures and test them using hand-held technology.

As a final note, Miguel's post-questionnaire simply confirmed his prior perspective for using hand-held CAS technology to make sense of mathematics. He wrote "I usually use technology to check the answers. I have used the internet before in an attempt to learn a concept, but I usually do not understand." Miguel, similar to others in the course, used hand-held CAS technology to gain a deeper insight into Plane Trigonometry. Nathan, on the other hand, reported "When I am no longer able to solve a problem using my own abilities." Nathan's outlook for the use of technology to make sense of mathematics was viewed as a secondary source rather than a tool to help guide his understanding. He continues by writing "it [hand-held CAS technology] is there as a support once I have reached my limits." He still seems to convey a struggle for using hand-held CAS technology to make sense of mathematics, while he understands the importance and ease for using the device. This was noted in question three of writing prompt III, where he was unable to complete the task.

Conclusion

The aim of the research was to investigate Plan Trigonometry students' *knowing* when to use hand-held CAS to make sense of mathematics at the colligate level. A secondary analysis for the study explored the students' position for using technology to make sense of mathematics. The results of the study indicated that the students had prior experience using technology to making sense of mathematics in PreK-5, middle grades, and secondary levels. The results also revealed that students had positive attitudes for using technology to make sense of mathematics. A major finding indicated that students used hand-held CAS when problems seemed difficult to learn, to speed the process for completing work, and verify pencil and paper solutions. Similar to Garner and Pierce (2005), students in the current research used hand-held CAS to make sense of mathematical phenomenon; however, students used these devices to compute straightforward calculations. Garner and Pierce (2005) suggest that students not use hand-held CAS for simple calculations. A goal for the study was to empower *all* students to use technology to make sense of mathematics

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Figure 1
Pre- and Post-Questionnaire – Created By Stewart, Thomas, and Hannah (2005).

Pre- and Post-Questionnaire
1. Technology does not improve my understanding of mathematics.
2. I waste a lot of time using technology
3. Technology helps me to visualize the problems.
4. I would prefer to use technology to do the calculations for me, so I can concentrate on the on understanding the concepts of the course.
5. I can solve mathematics problems using the computers even though I don't understand the theory
6. My answers are usually different from the answers that the technology gives me
7. I often check my answers using the technology.
8. I would like to learn more about the technology, so I can use it often.
9. I believe technology is the way to learn mathematics.
10. I hope to use my knowledge in computing in other courses when applicable.
11. My tutors are very supportive and encouraging in using the computer software.
12. I explore technology myself to learn more
13. I find it difficult to decide when to use the technology in doing mathematics problems.
14. Since I have been using technology, I have forgotten how to do the basic skills.
15. I like to use both technology and pencil and paper when working on mathematics problems.
16. I only use technology when I am stuck using pencil and paper for solving problems.
17. I find all the commands and instructions too difficult to remember.
18. Technology makes mathematics fun.
19. There is not enough support outside lecture time for using technology.
20. I believe technology gives me an unfair advantage in learning mathematics.
21. I think our mathematics labs are very helpful ad I enjoy going to the lab.
22. I would like to see the Professors use technology in lectures.

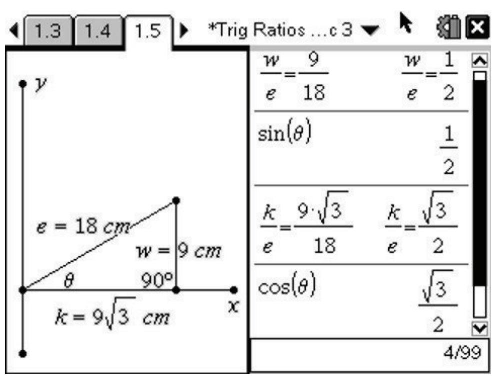

Figure 2
Question Three of Writing Prompt I - Exploring Trigonometric Functions.

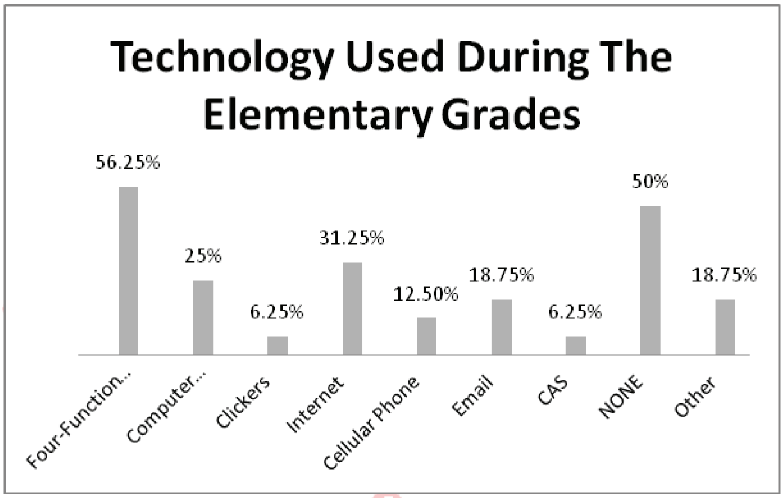

Figure 3
Technology Use Questionnaire – Students' Experience With Technology At The Elementary Levels

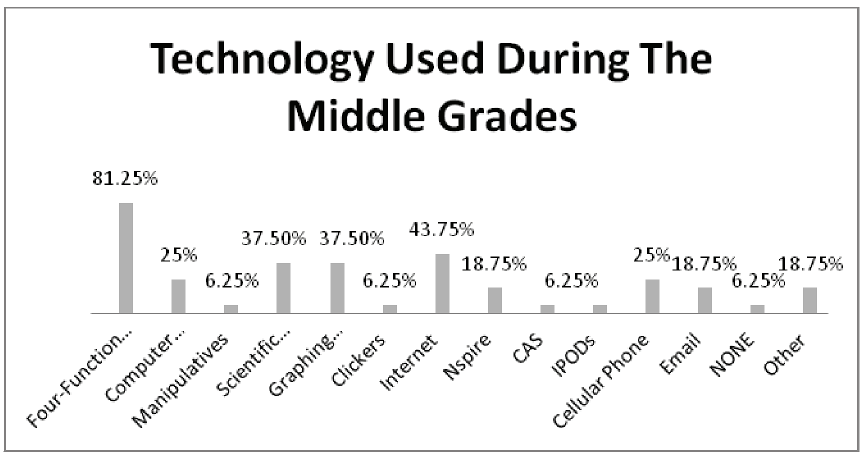

Figure 4
Technology Use Questionnaire – Students' Experience With Technology At The Middle Grades Levels

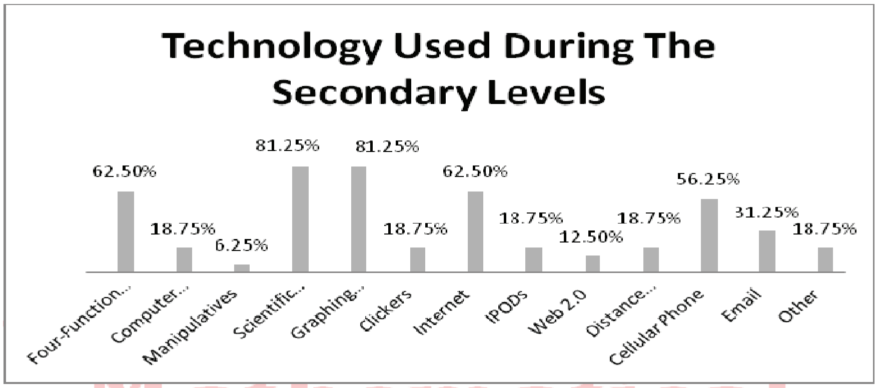

Figure 5
Technology Use Questionnaire – Student Experience With Technology At The Secondary Levels

	Agree		Neutral		Disagree	
	Pre	**Post**	**Pre**	**Post**	**Pre**	**Post**
Q1.	6.25	18.75	25.00	6.25	68.75	75.00
Q2	12.50	18.75	31.25	25.00	56.25	56.25
Q3	56.25	50.00	37.5	43.75	6.25	6.25
Q4	50.00	43.75	43.75	43.75	6.25	12.25
Q5	12.50	18.75	50.00	56.25	37.50	25.00
Q6	6.25	00.00	62.50	31.25	31.25	68.75
Q7	75.00	93.75	18.75	00.00	6.25	6.25
Q8	81.25	81.25	18.75	18.75	00.00	00.00
Q9	6.25	37.50	81.25	50.00	12.50	12.25
Q10	87.50	81.25	12.50	18.75	00.00	00.00
Q11	6.25	56.25	87.50	37.50	6.25	6.25
Q12	43.75	56.25	37.50	43.75	18.75	00.00
Q13	6.25	6.25	43.75	43.75	50.00	50.00
Q14	12.50	18.75	25.00	31.25	62.50	50.00
Q15	100.00	93.75	00.00	00.00	00.00	6.25
Q16	12.50	12.25	31.25	43.75	56.25	43.75
Q17	6.25	18.75	25.00	18.75	68.75	62.50
Q18	68.75	43.75	25.00	56.25	6.25	00.00
Q19	6.25	25.00	50.00	25.00	43.75	50.00
Q20	6.25	12.25	31.25	37.50	62.50	50.00
Q21	25.00	37.50	62.50	56.25	12.50	6.25
Q22	81.25	93.75	18.75	6.25	00.00	00.00

Figure 6
Pre- and Post- Questionnaire – Analysis In Percentages.

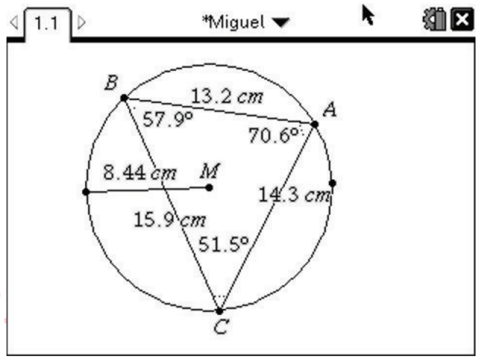

Figure 7
Miguel's Writing Prompt III Question One.

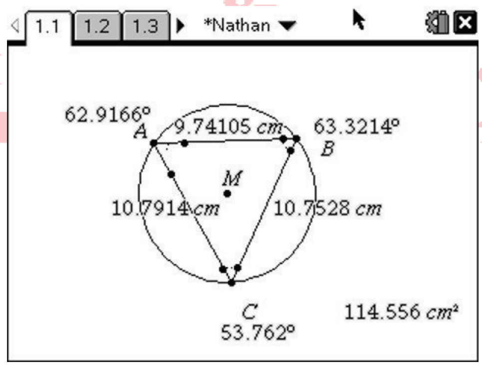

Figure 8
Nathan's Writing Prompt III Question One.

References

Asp, G. and McCrae, B. (2000). Technology-assisted mathematics education. In K. Owens & J. Mousley (EDs), *Research in Mathematics Education in Australia 1996-1999,* 123-160.

Ball, L., Stacey, K., and Leigh-Lancaster, D., (2001). Teaching Senior Maths with CAS: Major Issues for Curriculum, Assessment and Teaching. In C. Vale, J. Horwood & J. Roumeliotis (Eds.), 2001 *Mathematica odyssey* (Proceedings of the 38th annual conference of the Mathematical Association of Victoria, 2001, Melbourne, pp. 265-275), Melbourne: MAV.

Bossé, M. J., & *Nandakumar*, N. R. (*2004*). Computer algebra systems, pedagogy, and epistemology. *Mathematics and Computer Education*, 38, 298-306.

Burns, M. (2005). Looking at How Students Reason, *Educational Leadership*, 11, 26-31.

Connors, M.A. and Snook, K.G. (2001). The Effects of Hand-Held CAS on Student Achievement in a First Year College Core Calculus Sequence, *International Journal of Computer Algebra in Mathematics Education,* 8,99-114.

Connors, M.A. and Snoook, K.G. (2002) A Technology Tale: integrating hand-held CAS into a mathematics curriculum, *TheInstitute of Mathematics and its Applications,* 20,4, 171-190.

Creswell, J. W. (2003). Research design: Qualitative, quantitative, and mixed methods approaches. Thousand Oaks, California: Sage Publications, Inc.

Cuoco, A. & Manes, M. (September 2001). When memory fails. *Mathematics Teacher*, 94, 489-493.

Day, R. (1993). Algebra and technology. *Journal of Computers in Mathematics and Science Teaching*. 12, 29 – 36.

Garner, S. and Pierce, R. (2005). CAS: to everything there is a season. In Mathematics: Celebrating Achievement. (Proceedings of the 42nd annual Conference of the Mathematical Association of Victoria, CD-ROM). Melbourne: MAV.

Garofalo, J., Drier, H., Harper, S., Timmerman, M.A., and Shockey, T. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. *Contemporary Issues in Technology and Teacher Education,* 1, (1), 66-88.

Heck, A. (2001). Variables in computer algebra, mathematics and science. *The International Journal of Computer Algebra in Mathematics Education, 8*(3), 195-221.

Herget, W., Heugl, H., Kutzler, B., & Lehmann, E. (March 2001). Indispensable written calculation skills in a CAS environment. *Mathematics in School*, 30, 2-6.

Hillel, J. (1993). Computer algebra systems as cognitive technologies: Implications for the practice of mathematics education. In C. Keitel & K. Ruthven (Eds.), *Learning from computers: Mathematics education and technology* (pp. 18-47). Berlin: Springer-Verlag.

Guin, D. and Trouche, L. (1999). The complex process of converting tools into mathematical instruments: the case of calculators. *International Journal of Computers for Mathematical Learning 3,* 195-227.

Johnson, J. D. (2010). Prospective middle grades teachers of mathematics using hand-held CAS technology to create rich mathematical task. *Journal of Mathematical Sciences and Mathematics Education.* 5, 42 – 54.

Patton, M. Q. (2002). Qualitative research and evaluation methods (3rd Ed.). London: Sage Publications.

Pierce, R., and Stacey, K. (2001). Reflections on the changing pedagogical use of computer algebra systems: Assistance for doing or learning mathematics? *Journal of Computers in Mathematics and Science Teaching*, 20, 143-161.

Pierce, R. and Stacey, K. (2002). Monitoring effective use of computer algebra systems. In B. Barton, K.C. Irwin, M. Patrick, Pfannkuck, and M.O.J. Thomas (Eds.), *Mathematics Education in the South Pacific. Proceedings of the 25th AnnualConference of the Mathematics Education Research Group of Australasia* (pp. 575-582). Sydney: MERGA.

Pierce, R., and Stacey, K. (2004). Monitoring progress in algebra in a CAS active context: Symbol sense, algebraic insight and algebraic expectation. *International Journal for Technology in Mathematics Education*, 11, 3 – 11.

Stewart, S., Thomas, M. O. J., Hannah, J. (2005). Towards student instrumentation of computer-based algebra systems in university courses. *International Journal of Mathematical Education in Science and Technology.* 36, 741 – 749.

Vlachos, P., & Kehagias, A. (2000). A computer algebra system and a new approach for teaching business calculus. *The International Journal of Computer Algebra in Mathematics Education*, 7, 87 – 103.

Wiersma, W. (2000). Research methods in education: An introduction. Boston: Allyn and Bacon.

Zand, H., & Crowe, W. (2001). *Aspects of the influence of technology on teaching mathematics in the last decades.* In W. Yang, S. Chu, Z. Karian, & G. Fitz-Gerald, (Eds.), Proceedings of the 6th Asian Technology Conference in mathematics, Melbourne (pp. 77-86). Blacksburg, VA: ATCM.