Using the Cognitive Styles to Explain an Anomaly in the Hierarchy of the van Hiele Levels

Steve C. Perdikaris, Ph.D. †

Abstract

The hierarchy of levels is an important property in the van Hiele level theory of geometric thinking. The purpose of this work is to offer an exegesis of an anomalous result that appeared in the hierarchy in a three-dimensional geometry test. Participants were 50 students in three groups who had the same mathematical background. Some students did not fit the hierarchical model since they showed a better acquisition of level 3 than of Level 2, that is, they answered higher level items better than lower level ones. The two basic cognitive style dimensions are used to explain this peculiar result. This approach suggests the need for further empirical investigation to refute or support (never absolutely verify) the proposed exegesis.

Background

The van Hiele level theory of geometric thinking (Freudenthal, 1973; van Hiele, 1986; Wirszup,l976) contains important psychological and pedagogical concepts and is characterized by a striking simplicity. It provides a useful empirically-based description of what are likely to be relatively stable and qualitatively different levels of geometric understanding in students (Schoenfeld, 1986). Its epistemological route is a hierarchical procedure of five levels in Euclidean geometry that has significant implications for teaching, curriculum development and teacher training (Yazdani, 2007).

Thinking often reflects the enlightaned doubts and views of Plato and Aristotle whose philosophy was really devoted to serious thinking and in the largest sense is, according to John Dewey, "a general theory of education" (Weber, 1960). It also reflects the imprecision (vagueness and ambiguity) that is intrinsic in natural language (Perdikaris, 2011). This theory is reiforced by William T. Harris' stages of the development of thought where he uses Hegel's dialectic as the movement of thought towards a fuller knowledge (Weber, 1960).

The levels are situated not in the geometric subject matter but in the thinking of students and the transitions across the levels happen under the influence of geometric content and teaching methods (van Hiele, 1986). In Aristotelian fashion the levels are cumulative, that is, each one adds something to the previous one. What is implicit at one level becomes explicit at the next level since the way of learning at one level is verbalized and becomes the subject matter at the next level. Thus, each level is a metatheory of the previous one (Freudenthal, 1973).

The prolegomena suggest that the van Hiele level theory of geometric thinking is an example of a current theory that may be used as a framework for research in mathematics education that emphasizes goal-driven understanding.

A brief description of the levels as suggested by Hoffer (1985) is as follows:

- Level 1 (Visualization). Figures are judged by their appearance as a whole without regard to properties of their components.
- Level 2 (Analysis). The student begins to discern the properties of figures; figures are recognized as having parts and are recognized by their parts.
- Level 3 (Inference). The student logically orders the properties of concepts and figures, and uses one-step deduction.
- Level 4 (Deduction). The student can construct proofs, understand the role of axioms and definitions, and supply the reasons for steps in a proof.
- Level 5 (Rigor). The student can understand the formal aspects of deduction and can interrelate different axiomatic systems of proof.

An important property of the van Hiele levels is their hierarchy. The students must master one level of geometric thinking before proceeding to the next level. The hierarchical nature of levels is the skeleton of the theory and is based on Plato's ontological hierarchy of the epistemological route of five stages: tangible objects, elements, geometric solids, numbers and ideas. In the hierarchy of the van Hiele levels, a particular level is an outside to levels below it, and an inside to levels above it. Therefore, the status of a given level changes as one passes through in either the upward or the downward direction. This hierarchy is reinforced by Alfred N. Whitehead's stages of mental growth (Weber, 1960) and has been verified by several researchers (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1988; Gutierrez, Jaime & Fortuny,1991; Senk,1989; Usiskin, 1982).

Gutierrez et al.(l99l) evaluated the van Hiele levels of students' thinking by a spatial geometry test in three-dimensional geometry. The test was administered to a sample of 50 students in three groups of 21, 20 and 9 students who had the same mathematical background. They observed that within each group, the higher the level, the lower the degree of acquisition which agrees with the hierarchical structure of the levels. However, some students did not fit the hierarchical model because their degree of acquisition of Level 3 was higher than the degree of acquisition of Level 2. This means that they answered higher level items better than lower level ones. Other researchers (Aoyama, 2007; Mayberry, 1983; Perdikaris, 1996; Usiskin, 1982) observed similar anomalies. Gutierrez et al.(l99l) proposed that it is necessary to study this peculiarity in depth but no attempt to investigate it appeared in literature so far. This work offers an exegesis of this anomalous phenomenon using the basic cognitive styles.

Methods and Procedures

The term "cognitive style" or "thinking style" can be understood as a student's habitual preference of perceiving, thinking and remembering information. It refers to the manner in which thought is used, not to the power of thought. Tennant (1988) defines cognitive style as a student's "characteristic and consistent approach to organizing and processing information." It is a fairly fixed student characteristic mode of functioning and a key concept in education. Riding and Cheema (1991), after surveying the research on cognitive styles, concluded that there exist two basic orthogonal cognitive style dimensions which they termed Wholist-Analytic (W-A) and Verbal-Imagery (V-I). In Figure 1 the horizontal axis stands for the continuum which represents the W-A dimension while the vertical axis for the continuum which represents the V-I dimension.

The W-A dimension describes how a student habitually organizes and structures information in parts or wholes (Peterson and Deary, 2006). The relationship between parts and wholes is a. controversial issue in philosophy and is traced not only to ancient Greek philosophy but also to the older Chinese philosophy (Bahm, 1972). The whole and its parts are compatible, that is, are things of the same sort and their relations are central to understand learning and teaching.

Wholists perceive information globally and retain an overall view of information. They prefer to learn in groups, interact in the classroom and deal with open-ended problems where the emphasis is on variety and originality of responses (Riding and Cheema, 1991). Analytics perceive information in its component parts. They decompose a whole into a family of parts somehow simpler than the whole from which they were extracted and attempt to infer the properties of the whole from the properties of the parts. Analytics prefer problems that require one correct answer and greater logical ability. They are usually better to more independent approaches to learning (Riding and Cheema, 1991).

A student can use either the wholist or the analytic style but he has the tendency to use preferentially one of the two. The wholist style does not oppose the analytic style and vice versa, but the two styles of representation are complementary. In a proof by reduction ad absurdum, for example, a student is working analytically reasoning step by step, and arrives at a contradiction. Then working as a wholist grasps that the meaning of the whole process is the falsity of the assumption and the truth of the statement follows.

The V-I dimension describes student's mode of information representation in memory during thinking and affects the processing of information (Sternberg and Zhang, 2001). Information processing controls the flow of information into and out of working memory, and deals with the limitations imposed by the working memory capacity and the ways to overcome these limitations. Capacity refers to the amount of working memory, or attention span, that is required by received information.

Verbalizers represent information in words or verbal associations. They are better in details of dealing with actions and abstractions (Riding and Cheema, 1991). Imagers represent information in mental pictures. Their processing style is dominated by images. They are better on spatial and directional information and their representations have both pictorial quality and its associated wholeness (Riding and Cheema, 1991).

A student can use either the verbal or the imagery style but he has the tendency to use preferentially one of the two. The verbal style does not oppose the imagery style and vice verca, but the two styles of representation are complementary. A visual shape activates a definition of a geometric (abstract) entity. In defining the rectangle, for example, a student working as an imager sees the rectangle as a visual shape, i.e. like a door. Then working as a verbalizer creates a definition of the rectangle using the property "opposite sides parallel" of the rectangle which is the source from which the other properties spring. That is, a rectangle is a parallelogram (a quadrilateral in which opposite sides are parallel) with right angles.

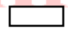
Research on cognitive styles suggests that the two orthogonal cognitive style dimensions are independent of one another, that is, the position of a student on one dimension does not affect his position on the other (Riding and Dyer, 1983). Thus, a student may be wholist and also imager, another may be analytic and imager or another may be analyric and verbalizer, while someone else may be verbalizer and wholist. The position of a student on the two orthogonal cognitive style dimensions can be measured by the Cognitive Style Analysis (CSA) which is a compiled computer-presented test (Riding, 1991).

The cognitive style is a potent variable which affects student's thinking, learning and academic development. Since it holds a great promise for understanding student behavior it should be interesting and useful to link up the basic cognitive styles with the van Hiele levels of geometric thinking (Perdikaris, 1997). Each quadrant in Figure 1 corresponds to the students who have achieved complete acquisition of a specific level. Notice that only the first four levels are considered since Level 5 does not appear in school geometric study.

In the first quadrant are the students who, working as wholist-imagers, have achieved complete acquisition of Level 1 which is characterized by student perception of geometric figures in their totality as entities according to their appearance. In the second quadrant are the students who, working as imager—analytics, have achieved complete acquisition of Level 2 which is characterized by student perception of properties of geometric figures and recognition of figures by their properties. In the third quadrant are the students who, working as analytic-verbalizers, have achieved complete acquisition of Level, 3 which is characterized by student ordering of the properties and the creation of a definition of the given figure. In the fourth quadrant are the students who, working as verbalizer-wholists, have achieved complete acquisition' of Level 4 which is characterized by student ability to grasp the significance of deduction as a means for constructing proofs.

The cycle in Figure 1 depicts a relational and at the same time dynamic field. It is a relational field since the cognitive styles in any two consecutive levels have a style in common. This means that these cognitive styles are not totally different but are partially related. It is also a dynamic field since there are transitions between consecutive levels which means that student thinking is characterized by frequent transitions between consecutive levels (Burger and Shaughnessy, 1986; Gutierrez et al., 1991). Students may move back and forth between consecutive levels in order to achieve complete acquisition of the higher level. The backward transition happens because students encounter special difficulties and try to get help from the methods of the lower level.

The back and forth transitions follow Hegel's dialectic (Fox, 2005). The term "dialectic" owes much of its prestige to its role in the philosophy of Socrates and refers to the analysis of meaning and the modes of reasoning. It consists of an assertion (thesis) which meets with its contradiction or negation (antithesis) and the tension between these opposites is resolved by absorption into a larger whole (synthesis).

The following example, taken from Crowley (1987), will help one to illustrate the above theoretical considerations. Consider responses to the questions "What type of figure is this?  How do you know?" Students at each level are able to respond "rectangle" to the first question. (If a student does not know how to name the figure, he or she is not at Level 1 for rectangles.) Examples of level-specific responses to the second question are given below. In parenthesis is a brief explanation of the way the statement reflects the assigned level.

<ins>Level 1:</ins> "It looks like one" or "Because it looks like a door." (The answer is based on a visual model.)

<ins>Level 2:</ins> "Four sides, closed, two long sides, two shorter sides, opposite sides parallel, four right angles..." (Properties are listed: redundancies are not seen.)

<ins>Level 3:</ins> "It is a parallelogram with right angles." (The student attempts to give a minimum number of properties. If queried, she would indicate that she knows it is redundant in this example to say that opposite sides are congruent.)

<ins>Level 4:</ins> "This can be proved if I know this figure is a parallelogram and one angle is a right angle." (The student seeks to prove the fact deductively.)

The cycle in the example above depicts a relational field. For example, in Level 2 the student working as an imager recognizes the rectangle in reference to a visual shape, i.e., "Because it looks like a door." Also working as an analytic recognizes it by its properties, that is, "Four sides, closed, two long sides, two shorter sides, opposite sides parallel, four right angles... "In Level 3 the student -working as an analytic recognizes the rectangle by a minimum number of properties where one of them is the source from which the others spring. This is the property "opposite sides parallel" that helps the student who also works as a verbalizer to create a definition of the rectangle, that is, "It is a parallelogram (a quadrilateral in which opposite sides are parallel) with right angles." It is obvious that the cognitive styles in Level 2 and Level 3 have the analytic style in common, which means that they are partially related. Similarly, the cognitive styles in any two consecutive levels are partially related. Hence, the cycle is a relational field.

The cycle in the example above is also a dynamic field. For example, the transition from Level 2 to Level 3 can be explained the same way as in Herbert Spencer's example of Hegelian dialectic (Weber, 1960). A student at Level 2 seeks to attain Level 3, that is, he tries to find a definition of the rectangle (thesis). This is problematic because it carries more information than usual in Level 2. The student is forced to give up before reaching the complexity structure that is expected in Level 3 and returns to Level 2 for help (antithesis). He must aim at something else because to aim at Level 3 directly is often to miss it. His target can be the property of the rectangle "opposite sides parallel" since it is requisite in a definition of parallelogram (a quadrilateral with opposite sides parallel) which is used to define the rectangle as "a parallelogram with right angles." Thus, pursuing ends other than Level 3 the attainment of Level 3 appears unbidden (synthesis). A similar treatment can be offered for the transitions from Level 1 to Level 2 and from Level 3 to Level 4. Hence, the cycle is a dynamic field.

Results and Conclusions

The hierarchy is derived from some primitive notions: an observer and his environment, an observed object and its environment and an interaction between the observer and the object. It has received substantial attention in research since it provides researchers with a convenient framework through which to focus their attentions in attempting to understand the processes of learning. Gagne (1963), using his hierarchical model, sought to validate the hypothesis that "an individual will not be able to learn a particular topic if he has failed to achieve any of the subordinate topics that support it." According to Yazdani (2007), the van Hiele levels "make a significant advance beyond the use of basic Gagne learning hierarchies because of their identification of qualitatively distinct levels and the role of reification in progressing from one level to the next."

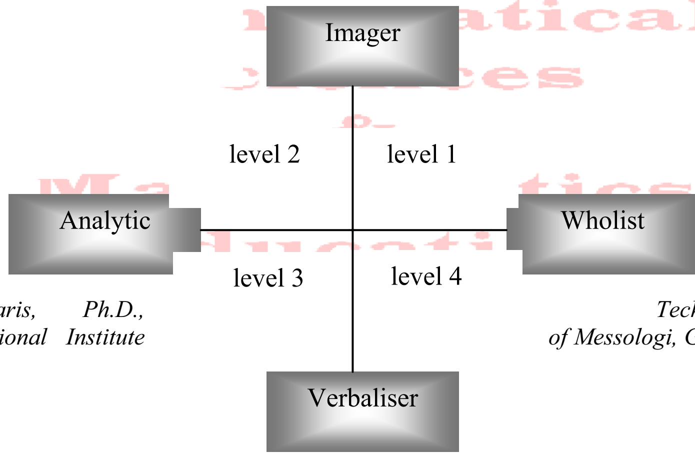
The cognitive styles are used as mediators of student geometric performance and as a tool to understand anomalies in the hierarchy of geometric development. They are variables affecting the student's academic choices and development, student learning, teaching and interactions in the classroom (Witkin, 1976).

The students who did not fit the hierarchical model because their degree of acquisition of level 3 was higher than the degree of acquisition of Level 2 were students who had the tendency to use preferentially the analytic and verbal cognitive styles. This suggested exegesis of the phenomenon is the hypothesis. It is instigated by the fact that the geometric subject matter involved in Level 3 has cognitive demands that fit their analytic and verbal styles. Students ordinarily are expected to do better in geometric subject matter in which cognitive demands fit their cognitive styles.

This hypothesis must be submitted to the test of experience using a hypothetico-deductive (hypothesis-prediction-experiment) method like Polya's

(1981) "guess and test." A prediction of the hypothesis is that the measure of students' position on the basic cognitive style dimensions is higher on the verbal axis than on the imager axis. Then one looks for the opposite of the prediction in order to refute or support (never absolutely verify) the hypothesis. One way to do this is to repeat the experiment in the work of Gutierrez et al.(1991) and apply Riding's (1991) Cognitive Style Analysis (CSA) on it in order to measure students' position on the cognitive style dimensions. If the hypothesis is supported it may stimulate research to illuminate other intriguing problems and areas of vagueness in the van Hiele level theory of geometric thinking.

Figure 1
Cognitive Style Dimensions and the van Hiele Levels



† Steve C. Perdikaris, Ph.D., Technological Educational Institute of Messologi, Greece.

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