Utilizing Euler's Approach in Solving Konigsberg Bridge Problem to Identify Similar Traversable Networks in a Dynamic Geometry Teacher Education Environment: An Instructional Activity

Mohammad A. Yazdani, Ph.D.†

Abstract

The Königsberg problem is a significant problem in mathematics history. The River Pregel passes through the city of Konigsberg, there are two islands in the river. There are seven bridges that join the different parts of the city to the islands. The objective is to find out if it was possible to walk across all the bridges such that each bridge is crossed only once. Simplifying the problem as a mathematical network, Leonhard Euler demonstrated that the network is not traversable. His work was the beginning of Graph Theory and Topology.

We designed and implemented a series of class activities to explore Euler's approach to other related problems in a teacher education geometry environment. The CABRI Geometry II software was utilized to carry the construction of geometric figures and networks. Students were instructed to construct various floor plans and investigate if they are traversable by drawing their correspondents' networks and applying Euler's principle. Participants were highly engaged in these activities and exhibited perseverance in multiple trials of constructing figures and networks, discussing the outcomes and accomplishing novel discoveries.

Introduction

Konigsberg was the capital of East Prussia. At the present it is in Russia territories and known as Kaliningrad. The River Pregel passes through the city There are two islands in the river. There are seven bridges that connect the different parts of the city to the islands.

Figure 1
Google Maps Satellite View of Konigsberg

Figure 2
Google Maps Satellite View of Konigsberg

Figure 3
Konigsberg

The people of Konigsberg used to walk through the city and islands via these seven bridges. They were wonder if it was possible to walk across all the bridges such that each bridge is crossed only once.

Leonhard Euler (Swiss mathematician, 1707-1783) became interested in solving this problem mathematically. Let define a few elementary definitions of the graph theory.

1. Network is a set of points and arcs where the ends of the arcs are elements of the set of points. The sides and vertices of a triangle or any other polygon form a network.
2. Traversable Networks are networks that one may starts at a point of the network and traverse, (in one connected journey) the complete network by passing each arc exactly one time.
3. Odd vertex is a vertex that is an end point for an odd number of arcs.
4. Arcs.
5. Even vertex is a vertex that is an end point for an even number of arcs.
6. In an even vertices network such as polygon, one may starts at any

vertex and traverses the network.
7. In an odd vertices network, one may start at one odd vertex and end up at the other odd vertex.

Euler found out that any traversable network has at most two odd vertices. By representing the problem as a network with vertices and arcs he demonstrated that there more than 2 odd vertices in the network and therefore the network is not traverse.

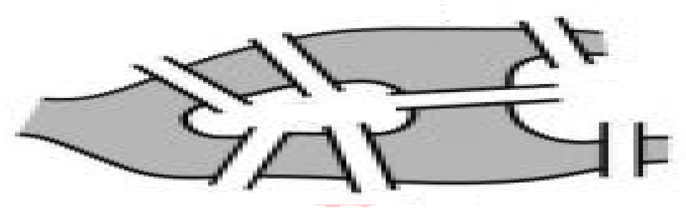

Figure 4
A Sketch of the Pregel river and islands created Paint

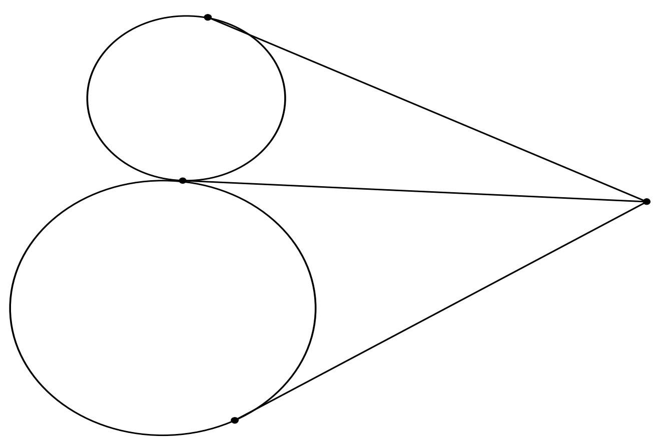

Figure 5
A Network of the Pregel river and islands created by geometric sketchpad

Inspiring by Königsberg problem we designed the series of floor plans and asked the students to apply Euler’s findings and determine if they were able to traverse through each unit by passing through each door not more than once. The followings are samples of that activity.

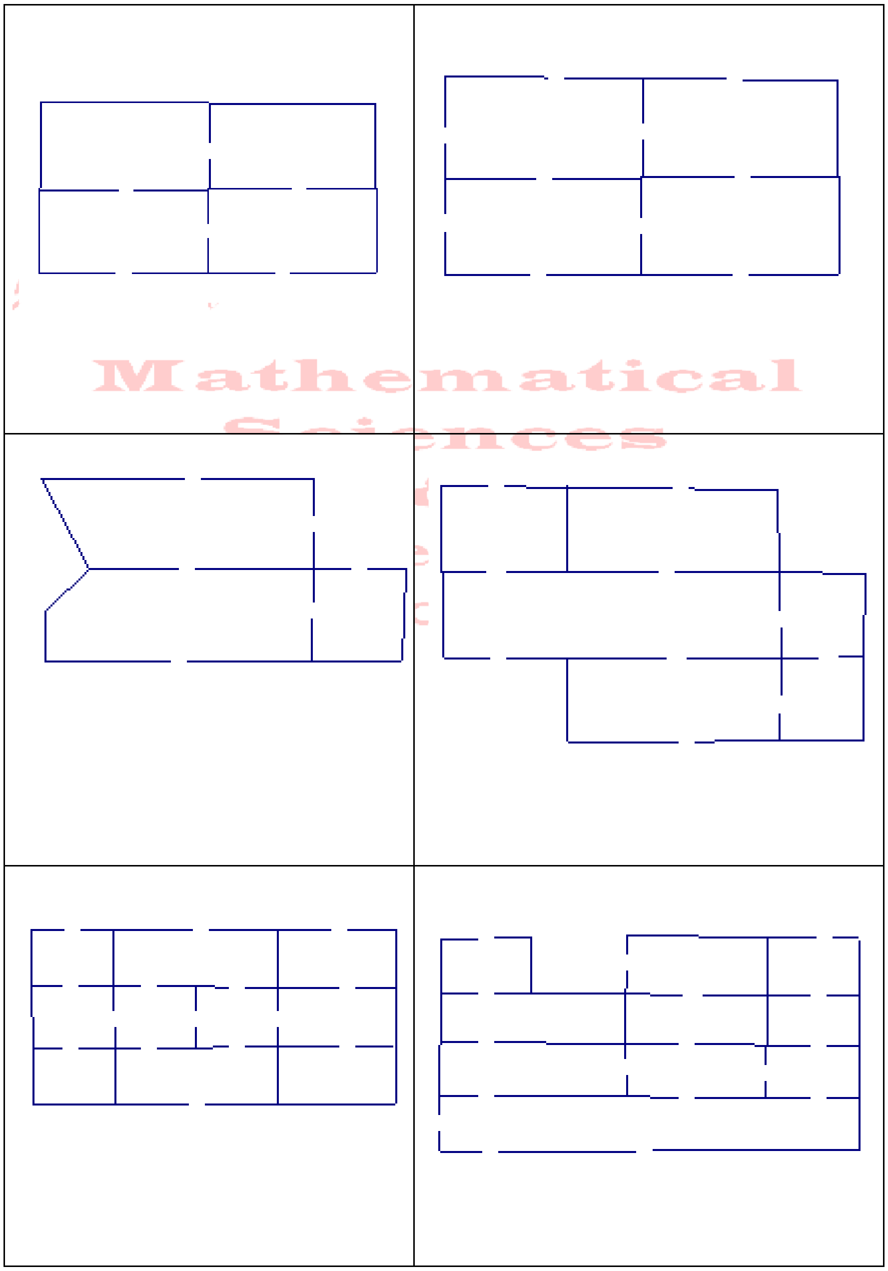

Figure 6
Samples of Floor plans activity

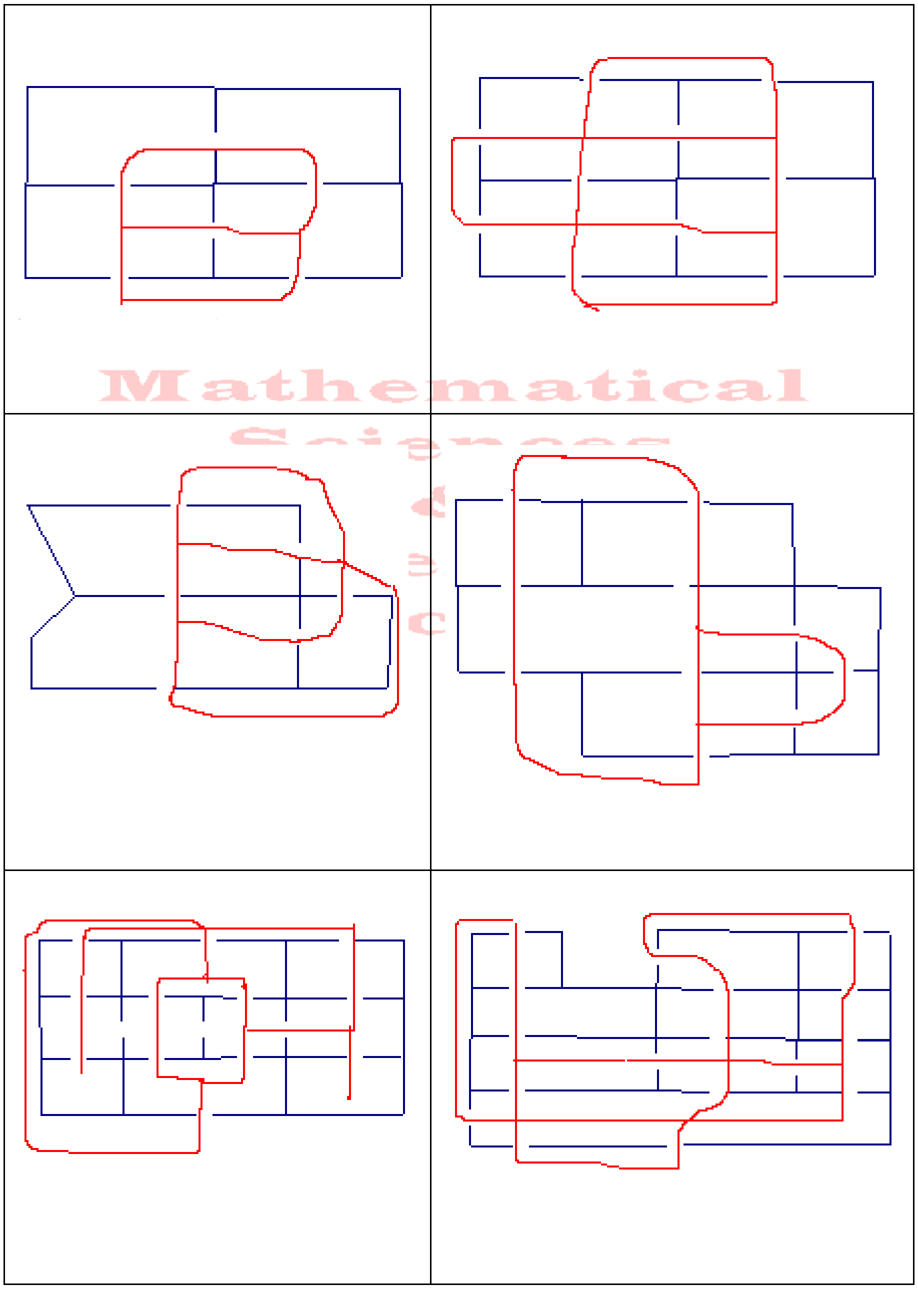

Figure 7
Constructing the network and vertices for each floor plan

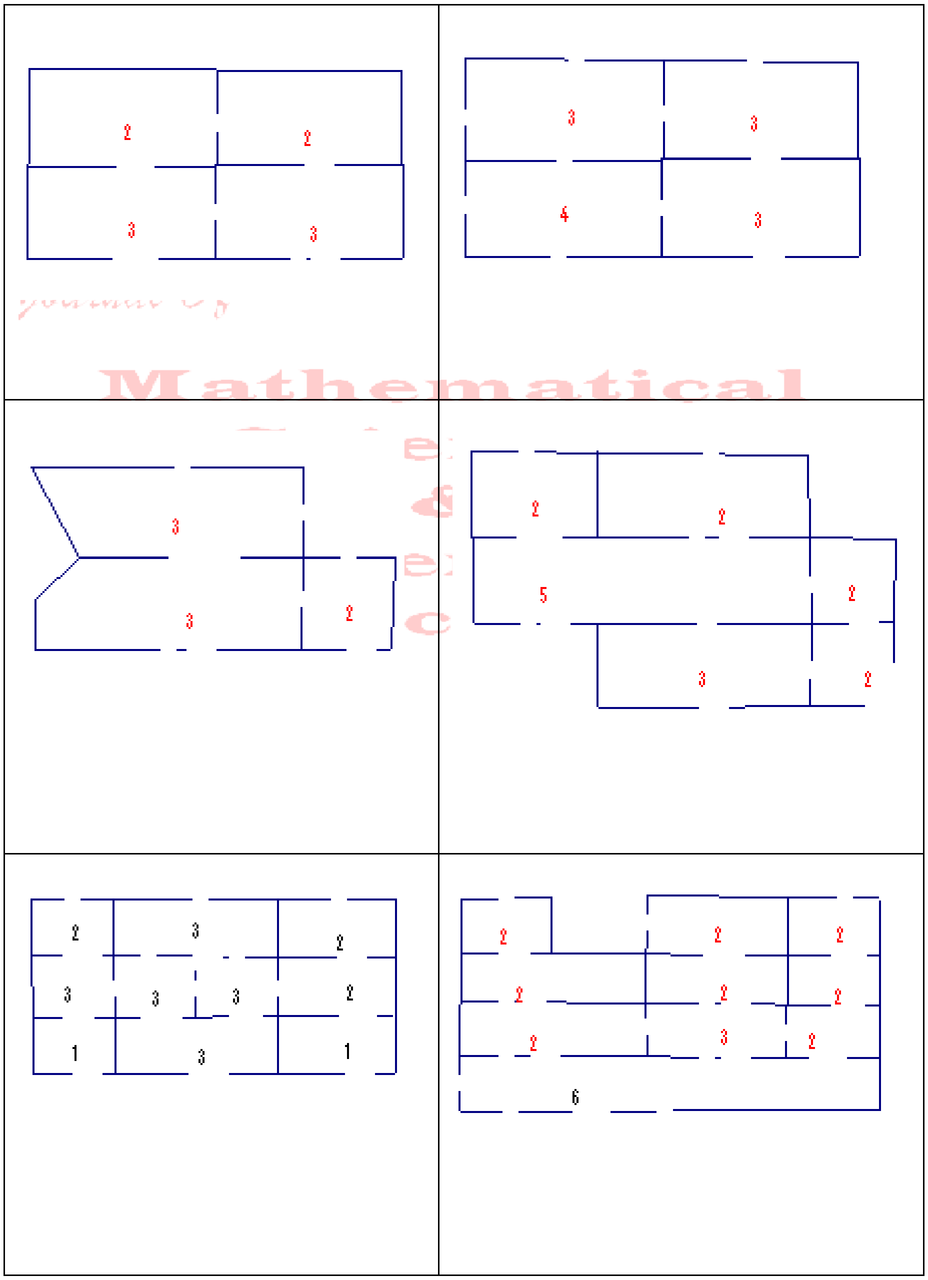

Figure 8
Counting the number of entries to each room (counting the number of the arcs going to each vertex)

Engagement

Our observations indicated that the students were very engaged with this class activity and many of them went beyond what was required..

Motivation

Some students were motivated and utilized CABRI Geometry II software and designed their own floor plans. They also constructed a network for each floor plan. Some of their floor plans and networks were rather complicated and challenging.

Perseverance

Several students posed their own extension questions, such as, “How do we design several floor plans for a two or three story houses with a basement such that we traverse through each unit of each floor by passing through each door once.

Conclusion

Students were engaged in this technology-enabled activity, many seeking additional resources and spending more time than usual on this activity. The activity was motivating as students searched for possible more complicated floor plans. The attitude to create their own floor plan designs and explore beyond the initial activity indicated that student perception of mathematics can be extended to pleasant and creative activities for K-12 curriculum.

Using CABRI Geometry II software allowed students to design and compare multiple examples of floor plans, as well as, it placed them in a self-directive role of investigation of Euler’s path.

†*Mohammad A. Yazdani, Ph.D.* University of West Georgia, USA

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Google Maps, https://maps.google.com/