Application of the Centroid Technique for Measuring Learning Skills

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Abstract

In earlier papers we have developed a fuzzy model for the process of learning and we have used the total possibilistic uncertainty for measuring student groups' learning skills. In this paper based on the above model we apply the centroid defuzzification technique as an alternative assessment method. These two methods, which treat differently the idea of the students' performance, are compared to each other and examples are presented illustrating the differences between them. Fuzzy methods for the students' individual assessment are also studied.

Introduction

The concept of learning is fundamental to the study of human cognitive action. But while everyone knows in general what learning is, the understanding of its nature has proved to be complicated. This basically happens because it is very difficult for someone to understand the way in which the human mind works, and therefore to describe the mechanisms of the acquisition of knowledge by the individual. The problem is getting even harder by taking into consideration the fact that these mechanisms, although they appear to have some common general characteristics, they actually differ in details from person to person.

There are very many theories and models developed by psychologists and education researchers for the description of the mechanisms of learning. Voss (1987) adopted an argument raised much earlier by Ferguson (1956) and others that learning is a specific case of the general class of transfer, i.e. the use of already existing knowledge to produce new knowledge. Accordingly, Voss argued that learning basically consists of successive problem – solving activities, in which the input information is represented of existing knowledge, with the solution occurring when the input is appropriately interpreted. The process involves the following stages: *Representation* of the input data, *interpretation* of this data in order to produce the new knowledge, *generalization* of the new knowledge to a variety of situations and *categorization* of the generalized knowledge.

More explicitly the representation of the stimulus input is relied upon the individual's ability to use contents of his (her) memory in order to find information that will facilitate a solution development. Learning consists of developing an appropriate number of interpretations and generalizing them to a variety of situations. When the knowledge becomes substantial, much of the process involves categorization, i.e. the input information is interpreted in terms of the classes of the existing knowledge. Thus the individual becomes able to relate new information to his (her) knowledge structures that have been variously described as schemata, or scripts, or frames.

The knowledge that students have about various concepts is usually imperfect, characterized by a different degree of depth. From the teacher's point of view on the other hand there exists vagueness about the degree of students' success in each stage of the learning process. All these gave us the impulsion to introduce principles of fuzzy logic in order to achieve a better and more realistic representation of the process of learning. In fact, in earlier papers we have developed a fuzzy model for learning (Voskoglou 1999) and we have used the total possibilistic uncertainty in assessing student groups' learning abilities (Voskoglou 2009). In this paper we shall use the centroid defuzzification technique that will enable us to compare the learning skills of student groups' at each stage of the learning process. We shall also study methods of students' individual assessment. For general facts on fuzzy sets and logic we refer freely to the book of Klir and Folger (1988).

The fuzzy model

Our fuzzy model developed in Voskoglou (1999) is based on the above mentioned Voss's theory about learning and its main ideas are the following:
Let us consider a group of n students, $n \geq 2$, during the learning process of a subject matter in the classroom. We denote by A_i , $i=1,2,3$, the stages of representation/interpretation, generalization and categorization respectively, and by a, b, c, d, and e the linguistic labels of negligible, low, intermediate, high and complete success respectively at each of the A_i*'s*. We notice that, in order to make our model technically simpler, we have considered the stages of representation and interpretation of the Voss's approach as a unique stage. This is close to the reality, since representation is actually an introductory stage of the learning process.

Set $U=\{a, d, c, d, e\}$ and denote by n_{ia}, n_{ib}, n_{ic}, n_{id} *and* n_{ie} the numbers of students that have achieved negligible, low, high and complete success at state A_i respectively, $i=1,2,3$, In order to represent the A_i*'s* as fuzzy subsets of U. we define the membership function m_{A_i} in terms of the frequencies, i.e. by $m_{A_i}(x) = \frac{n_{ix}}{n}$ for each x in U. Then we can write $A_i = \{(x, \frac{n_{ix}}{n}) : x \in U\}$.

A *student's profile* during the learning process is defined to be an ordered triple of the form

$s = (x, y, z)$, where x, y and z are elements of U that denote the student's success at the stages A_1, A_2 and A_3 respectively. The rest of our model involves the representation of all possible students' profiles as a fuzzy subset of U^3 (through the proper definition of the membership degree m_s of each profile s) and the calculation of the possibilities of all profiles by the well known formula $r_s = \frac{m_s}{\max\{m_s\}}$, where $max\{m_s\}$ denotes the maximal value of m_s, for all s in U^3.

In other words r_s is the "relative membership degree" of s with respect to the membership degrees of the other profiles.

In this way we obtain a qualitative view of the students' performance during the learning process of a subject matter in the classroom. This is reinforced by Shackle (1961), and many others after him, who argues that human cognition can be formalized more adequately by possibility rather, than by probability theory. We recall the probability for fuzzy data is defined by $p_s = \frac{m_s}{\sum_{s \in U^3} m_s}$, which gives that $p_s \leq r_s$ for all s in U^3. This is compatible to the common logic, since whatever it is probable it is also possible, but whatever is possible need not be very probable.

We must emphasize that in our model we considered the process of learning a subject matter in the classroom only and not the process of learning by the individual in general. In fact, learning is a very composite and complicated action of the human mind, whose stages could be reached out of the class, or in a next class, or even during sleeping! Therefore it is inevitable for someone to put some restrictions in order to attempt a mathematical description of the process of learning, even when using principles of fuzzy logic for this purpose.

A basic principle of the information theory states that the amount of information obtained by an action can be measured by the reduction of uncertainty that results from the action. Thus a measure of a student group's uncertainty can be also adopted as a measure of its performance. In fact, the lower is a group's uncertainty after the learning process, which indicates a greater reduction of it during the process, the better is its performance.

In Voskoglou (2009) we have used a student group's *total possibilistic uncertainty* (i.e. the sum of *strife* and *non specificity*) as a measure of its performance during the learning process. The above measure is calculated in terms of the ordered possibility distribution of the student group (for more details the reader may look also at Voskoglou 2012b). Other measures of uncertainty that are commonly used in fuzzy logic involve the *total probabilistic uncertainty***,** i.e. the classical *Shannon's entropy* expressed in terms of the Dempster-Shafer mathematical theory of evidence for use in a fuzzy environment (e.g. see Voskoglou 2012a) and the *ambiguity* which is a generalization of the Shannon's entropy in possibility theory that captures both strife and non specificity (e.g. see Perdikaris, 2012).

The Centroid defuzzification technique

Another popular technique of producing a quantifiable result from fuzzy data (*defuzzification*) is the *centroid method*, in which the coordinates of the centre of gravity of the graph of the membership function involved provide an alternative measure of a group's performance (e.g. see van Broekhoven and De Baets 2006). The application of the 'centroid method' in practice is simple and evident and, in contrast to the measures of uncertainty, needs no complicated calculations in its final step. Further this method enables one to compare a student group's performance at each stage of the learning process. The techniques that we shall apply here have been also used in Subbotin et al. 2004, in Voskoglou and Subbotin 2012, etc.

Given a fuzzy subset $A = \{(x, m(x)): x \in U\}$ of the universal set U of the discourse with membership function $m: U \rightarrow [0, 1]$, we correspond to each $x \in U$ an interval of values from a prefixed numerical distribution, which actually means that we replace U with a set of real intervals. Then, we construct the graph F of the membership function $y=m(x)$.There is a commonly used in fuzzy logic approach to measure performance with the pair of numbers (x_c, y_c) as the coordinates of the *centre of gravity*, say F_c, of the graph F, which we can calculate using the following well-known from Mechanics formulas:

$$x_c = \frac{\iint_F x\,dxdy}{\iint_F dxdy}, \quad y_c = \frac{\iint_F y\,dxdy}{\iint_F dxdy} \qquad (1)$$

Concerning the learning process, we characterize an individual's performance as very low (*a*) if x $\in$ [0, 1), as low (*b*) if $x \in [1, 2)$, as intermediate (*c*) if $x \in [2, 3)$, as high (*d*) if $x \in [3, 4)$ and as very high (*e*) if $x \in [4, 5]$ respectively. Therefore, if $x \in [1, 2)$, then $m(x) = m(a)$, if $x \in [1, 2)$ then $m(x)= m(b)$ and so on. Thus, in this case the graph F of the corresponding fuzzy subset of U is the bar graph of Figure 1 consisting of five rectangles, say F_i, i=1,2,3, 4, 5 , whose sides lying on the x axis have length 1.

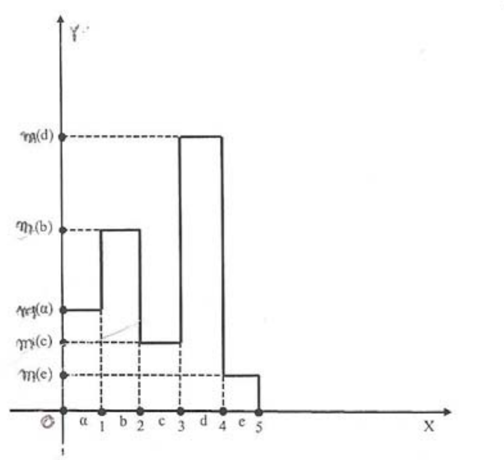

Figure 1: Bar graphical data representation

In this case $\iint\limits_F dxdy$ is the area of F which is equal to $\sum_{i=1}^{5} y_i$. Also $\iint\limits_F xdxdy$

$= \sum_{i=1}^{5} \iint\limits_{F_i} xdxdy = \sum_{i=1}^{5} \int_0^{y_i} dy \int_{i-1}^{i} xdx = \sum_{i=1}^{5} y_i \int_{i-1}^{i} xdx = \frac{1}{2} \sum_{i=1}^{5} (2i-1) y_i$, and

$\iint\limits_F ydxdy = \sum_{i=1}^{5} \iint\limits_{F_i} ydxdy = \sum_{i=1}^{5} \int_0^{y_i} ydy \int_{i-1}^{i} dx = \sum_{i=1}^{n} \int_0^{y_i} ydy = \frac{1}{2} \sum_{i=1}^{n} y_i^2$. Therefore formulas (1) are transformed into the following form:

$$x_c = \frac{1}{2}\left(\frac{y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5}{y_1 + y_2 + y_3 + y_4 + y_5}\right),$$

$$y_c = \frac{1}{2}\left(\frac{y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2}{y_1 + y_2 + y_3 + y_4 + y_5}\right). \quad (2)$$

Normalizing our fuzzy data by dividing each $m(x)$, $x \in U$, with the sum of all membership degrees we can assume without loss of the generality that $y_1+y_2+y_3+y_4+y_5 = 1$. Therefore we can write:

$$x_c = \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right),$$

$$y_c = \frac{1}{2}\left(y_1^2 + y_2^2 + y_3^2 + y_4^2 + y_5^2\right) \quad (3)$$

with $y_i = \dfrac{m(x_i)}{\sum_{x \in U} m(x)}$ ..

But $0 \leq (y_1-y_2)^2 = y_1^2+y_2^2-2y_1y_2$, therefore $y_1^2+y_2^2 \geq 2y_1y_2$,with the equality holding if, and only if, $y_1=y_2$. In the same way one finds that $y_1^2+y_3^2 \geq 2y_1y_3$, and so on. Hence it is easy to check that

$(y_1+y_2+y_3+y_4+y_5)^2 \leq 5(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2)$, with the equality holding if, and only if $y_1=y_2=y_3=y_4=y_5$.

But $y_1+y_2+y_3+y_4+y_5 = 1$, therefore $1 \leq 5(y_1^2+y_2^2+y_3^2+y_4^2+y_5^2)$ (4), with the equality holding if, and only if $y_1=y_2=y_3=y_4=y_5=\frac{1}{5}$. Then the first of formulas (3) gives that $x_c = \frac{5}{2}$. Further, combining the inequality (4) with the second of formulas (3) one finds that $1 \leq 10y_c$, or $y_c \geq \frac{1}{10}$ Therefore the unique minimum for y_c corresponds to the centre of mass $F_m(\frac{5}{2}, \frac{1}{10})$.

The ideal case is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Then from formulas (3) we get that $x_c = \frac{9}{2}$ and $y_c = \frac{1}{2}$. Therefore the centre of mass in this case is the point $F_i\left(\frac{9}{2}, \frac{1}{2}\right)$.

On the other hand the worst case is when $y_1=1$ and $y_2=y_3=y_4=y_5=0$. Then for formulas (3) we find that the centre of mass is the point $F_w\left(\frac{1}{2}, \frac{1}{2}\right)$. Therefore the "area" where the centre of mass F_c lies is represented by the triangle $F_w F_m F_i$ of Figure 2.

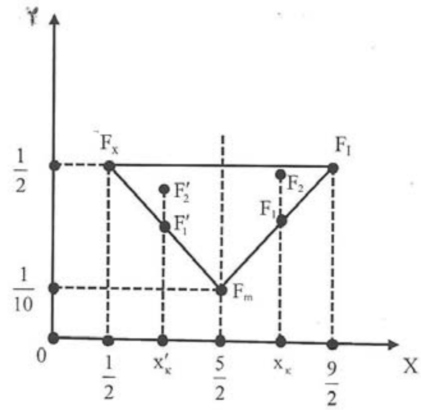

Figure 2: Graphical representation of the "area" of the centre of mass

Then from elementary geometric considerations it follows that the greater is the value of x_c the better is the group's performance. Also, for two groups with the same $x_c \geq 2{,}5$, the group having the centre of mass which is situated closer to F_i is the group with the higher y_c; and for two groups with the same $x_c < 2.5$ the group having the centre of mass which is situated farther to F_w is the group with the lower y_c. Based on the above considerations it is logical to formulate our criterion for comparing the groups' performances in the following form:

- *Among two or more groups the group with the biggest x_c performs better.*
- *If two or more groups have the same $x_c \geq 2.5$, then the group with the higher y_c performs better.*
- *If two or more groups have the same $x_c < 2.5$, then the group with the lower y_c performs better.*

The following example illustrates the use of the centroid technique in practice:

EXAMPLE: In this example we use the fuzzy data obtained by a classroom experiment performed some years ago with two groups of 35 and 30 students respectively of the School of Management and Economics of the Graduate Technological Educational Institute of Messolonghi, in Greece, when I was teaching the definite integral (see section 4 of Voskoglou 2009).

Let us denote by A_{ij} the fuzzy subset of U attached to the stage A_j, $j=1,2,3$ of the process of learning with respect to the group i, $i= 1,2$. Then the fuzzy data mentioned above can be written as follows

First group:

$A_{11} = \{(a, 0), (b, 0), (c, \frac{17}{35}), (d, \frac{8}{35}), (e, \frac{10}{35})\}$. $A_{12} = \{(a, \frac{6}{35}), (b, \frac{6}{35}), (c, \frac{16}{35}), (d, \frac{7}{35}), (e, 0)\}$ and

$A_{13} = \{(a, \frac{12}{35}), (b, \frac{10}{35}), (c, \frac{13}{35}), (d, 0), (e, 0)\}$

Second group:

$A_{21}=\{(a, 0), (b, \frac{6}{30}), (c, \frac{15}{30}), (d, \frac{9}{30}), (e, 0)\}$, $A_{22}=\{(a, \frac{6}{30}), (b, \frac{8}{30}), (c, \frac{16}{30}), (d, 0), (e, 0)\}$ and

$A_{23}=\{(a, \frac{12}{30}), (b, \frac{9}{30}), (c, \frac{9}{30}), (d, 0), (e, 0)\}$.

Applying the first of formulas (3) to the above fuzzy data one finds:

$x_{c11}= \frac{1}{2}(5.\frac{17}{35} + 7.\frac{8}{35} + 9.\frac{10}{35}) = \frac{231}{70} =3,3$ and $x_{c21}= \frac{1}{2}(3.\frac{6}{30} + 5.\frac{15}{30} + 7.\frac{9}{30}) = \frac{156}{60} =2,6$.

Therefore by our criterion the first group demonstrates a better performance at the stage of representation/interpretation.

Similarly one finds that

$x_{c12}= \frac{1}{2}(\frac{6}{35} +3.\frac{6}{35} +5.\frac{16}{35} +7.\frac{7}{35})= \frac{193}{70} \approx 2,757$ and $x_{c22}= \frac{1}{2}(\frac{6}{30} + 3.\frac{8}{30} + 5.\frac{16}{30})= \frac{110}{60} \approx 1,833$.

Therefore the first group demonstrates again a better performance at the sage of generalization.

Finally for the last sage of categorization on finds that

$x_{c13}= \frac{1}{2}(\frac{12}{35} +3.\frac{10}{35} +5.\frac{13}{35})= \frac{107}{70} \approx 1,528$ and $x_{c23}= \frac{1}{2}(\frac{12}{30} + 3.\frac{9}{30} + 5.\frac{9}{30}) = \frac{84}{60} = 1,4$. Therefore the first group demonstrates again a slightly better performance.

In concluding the performance of the first group was found to be better at all stages of the learning process. We observe also that the more advanced is a stage, the less good was the performance of each group. This was logically expected due to the obvious increasing difficulty of each stage

Notice that in Voskoglou 2009 we found that the total possibilistic uncertainty for the first group is 2,97 , while for the second one is 2,322. But this means that the second group demonstrates a better performance, which contradicts our previous conclusion! This is due to the fact that the above two approaches treat differently the idea of a group's performance. In fact, in the first case the student group's uncertainty during the learning process is connected to its capacity in obtaining the relevant information. In other words, in this case we are looking for the *mean group's performance*. On the other hand, in the case of the centroid technique the *weighted average* plays the main role, i.e. the results of the performance close to the ideal performance have much more weight than those close to the lower end. In other words, in this case we are mostly looking at the *quality* of the performance. Therefore some differences could appear in boundary cases. In concluding, it is argued that the combined application of these two approaches helps in finding the ideal profile of performance according to the user's personal criteria of goals and therefore to finally choosing the appropriate approach for measuring the results of his/her experiments.

Notice also that in Voskoglou 2009-10 we have developed a stochastic model by introducing a finite Markov chain on the stages of the learning process. This model is helpful in understanding the "ideal behaviour" of learners, in which they proceed linearly from representation to the final stage of categorization through the other stages of the process of learning. However, it has been observed that students take actually individual routes during learning. Therefore a qualitative study of all possible students' profiles becomes necessary for a deeper understanding of the mechanisms of learning, which is obtained through the use of our fuzzy model. On the other hand, the development of a fuzzy model in general has the disadvantage of depending on the researcher's personal criteria. In case of our model for learning for example, these criteria are involved in characterizing the students' performance in terms of a set of linguistic labels which are fuzzy themselves, in choosing the proper membership function and the proper defuzzification technique, etc. Therefore the use of our stochastic model as a tool for the validation of the fuzzy one seems to be a good solution in achieving a worthy of credit mathematical analysis of the process of learning (see also the book Voskoglou 2011).

3. Methods of individual assessment of students' learning skills

One of the main teachers' concerns is the assessment of their students' knowledge and aptitudes. In fact, our society demands not only to educate, but also to classify the students according to their qualifications as being suitable or not to carry out certain tasks or to hold certain posts. According to the standard methods of assessment, a mark, expressed either with a numerical value within a given scale (e.g. from 0 to 10) or with a letter (e.g. from A to F) corresponding to the percentage of a student's success, is assigned in order to characterize his/her performance. However, this crisp characterization, based on principles of the bivalent logic (yes-no), although it is the one usually applied in practice, it is not probably the most suitable to determine a student's performance. In fact, the teacher can be never absolutely sure about a particular numerical grade characterizing the student's abilities and skills. On the contrary, fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for this purpose.

Our fuzzy model for learning presented above can be used also (in a simplified form) for the students' individual assessment. In fact, if $n=1$, then from the definition of the membership function m_{A_i} it becomes evident that in each A_i, i = 1, 2, 3, there exists a unique element x of U with membership degree 1, while all the others have membership degree 0. Consequently there exists a unique student profile s with $m_s= 1$, while all the others have membership degree 0. For example, if $m_{A_1}(d)=1,\ m_{A_2}(c)=1,\ m_{A_3}(b)=1$, then $s = (d, c, b)$. In other words, each student is characterized in this case by a unique profile, which gives us the requested information about his/her performance.

A. Jones developed a fuzzy model to the field of Education involving several theoretical constructs related to assessment, amongst which is a technique for assessing the deviation of a student's knowledge with respect to the teacher's knowledge , which is taken as a reference (see Jones et al. 1986, Espin and Oliveras 1997). Here we shall present this technique, properly adapted with respect to our fuzzy model.

Let $X= \{A_1, A_2, A_3\}$ be the set of the stages of the learning process as they have been considered in section 2. Then a fuzzy subset of X of the form $\{(A_1, m(A_1)), (A_2, m(A_2)), (A_3, m(A_3)\}$can be assigned to each student , where the membership function m takes the values $0, 0.25, 0.5, 0.75, 1$ according to the level of the student's performance at the corresponding step. The teacher's fuzzy measurement is always equal to 1, which means that the fuzzy subset of X corresponding to the teacher is $\{(A_1, 1), (A_2, 1), (A_3, 1)$.

Then the *fuzzy deviation* of the student i with respect to the teacher is defined to be the fuzzy subset $D_i=\{(A_1,1-m(A_1)), (A_2, 1-m(A_2)), (A_3,1- m(A_3)\}$ of X. This assessment by reference to the teacher provides us with the ideal student as the one with nil deviation in all his/her components and it defines a relationship of partial order among students'. The following example illustrates this theoretical framework in practice.

EXAMPLE: The same experiment with that mentioned in the example of section 2 was repeated recently with a group of 35 students of the School of Techological Applications (future engineers) of the Technological Educational Institute of Patras, Greece. This time in assessing the students' individual performance by applying the A. Jones technique we found the following types of deviations with respect to the teacher:

$D_1 = \{(A_1, 0.75), (A_2, 0.75), (A_3, 1)\}$ (this type of deviation was related with *2* students)
$D_2 = \{(A_1, 0.5), (A_2, 1), (A_3, 1)\}$ (related with *7* students)
$D_3 = \{(A_1, 0.5), (A_2, 0.75), (A_3, 1)\}$ (related with *5* students)
$D_4 = \{(A_1, 0.5), (A_2, 0.75), (A_3, 0.75)\}$ (related with *4* students)
$D_5 = \{(A_1, 0.25), (A_2, 0.5), (A_3, 0.75)\}$ (related with *3* students)
$D_6 = \{(A_1, 0.25), (A_2, 0.25), (A_3, 0.5)\}$ (related with *6* students)

D_7 = {$(A_1, 0), (A_2, 0.5), (A_3, 0.75)$} (related with *1* student)
D_8 = {$(A_1, 0), (A_2, 0.5), (A_3, 0.5)$} (related with *2* students)
D_9 = {$(A_1, 0), (A_2, 0.25), (A_3, 0.5)$} (related with *1* student)
D_{10} = {$(A_1, 0), (A_2, 0.25), (A_3, 025)$} (related with *3* students)
D_{11} = {$(A_1, 0), (A_2, 0), (A_3, 0.25)$} (related with *1* student)

On comparing the above types of students' deviations it becomes evident that the students possessing the type D_3 of deviation demonstrate a better performance than those possessing the type D_1, the students possessing the type D_4 demonstrate a better performance than those possessing the type D_3 and so on. However, the students possessing the type D_1 demonstrate a better performance at the stage of generalization than those possessing the type D_2, who demonstrate a better performance at the stage of representation/interpretation. Similarly, the students possessing the type D_6 demonstrate a better performance at the steps of generalization and categorization than the student possessing the type D_7, who demonstrates a better performance at the step of representation/interpretation. In other words, the students' deviations define a *relationship of partial order* among the students with respect to their total performance.

Notice that each deviation D_i corresponds to a student's profile s_i, $i = 1, 2,\ldots, 11$. For example, the deviation D_1 corresponds to the student {$(A_1, 0.25), (A_2, 0.25), (A_3, 0)$}, whose profile is $s_1 = (b, b, a)$. Applying the same argument one finally finds the following profiles characterizing the students' performance in our experiment:

$s_1 = (b, b, a)$ (this profile is related with *2* students)
$s_2 = (c, a, a)$ (related with *7* students)
$s_3 = (c, b, a)$ (related with *5* students)
$s_4 = (c, b, b)$ (related with *4* students)
$s_5 = (d, c, b)$ (related with *3* students)
$s_6 = (d, d, c)$ (related with *6* students)
$s_7 = (e, c, b)$ (related with *1* student)
$s_8 = (e, c, c)$ (related with *2* students)
$s_9 = (e, d, c)$ (related with *1* student)
$s_{10} = (e, d, d)$ (related with *3* students)
$s_{11} = (e, e, d)$ (related with *1* student)

In other words, the A. Jones technique is actually equivalent to our method for the students' individual assessment. The only difference is that the former expresses the fuzzy data with numerical values, while the latter expresses it qualitatively in terms of the fuzzy linguistic labels of *U*.

Notice also that the teacher may put a target for his/her class and may establish didactic strategies in order to achieve it. For example he/she may ask for the deviation, say *D*, with respect to the teacher to be $0.25 \leq D \leq 0.5$, for all students and in all steps. In this case the application of the A. Jones technique could help the teacher to determine the divergences with respect to this target and hence to readapt his/her didactic plans in order to diminish these divergences.

4. Conclusions

The following conclusions can be drawn from those presented in this paper:

- Fuzzy logic, due to its nature of including multiple values, offers a wider and richer field of resources for assessing the students' performance than the classical crisp characterization does by assigning a mark to each student.
- In earlier papers we have developed a fuzzy model for the process of learning a subject matter in the classroom and we have used the total possibilistic uncertainty in assessing student groups' learning abilities. In this paper we used the centroid defuzzification technique that enabled us to compare the learning skills of student groups' at each stage of the learning process. These two assessment methods, which treat differently the idea of students' performance, were compared to each other and examples were given to illustrate the differences between them.
- Our fuzzy model can be also applied in a simplified form for the students' individual assessment. In this case a qualitative profile of the form (*x, y, z*) is assigned to each student, where *x, y and z* are fuzzy linguistic labels characterizing the degree of the student's success in each stage of the learning process.
- The A. Jones technique for assessing the deviation of a student's knowledge with respect to the teacher's knowledge can be also applied for measuring the students' learning skills on an individual basis. This technique is actually equivalent with our method, the only difference being that it expresses the fuzzy data with numerical values. However this approach is more helpful when the teacher puts a target for his/her class and establishes didactic strategies in order to achieve it.

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