Sixth Grade Mathematics Students: Expert-Novice Distinction of Area and Perimeter of the Rectangle

Haitham M. Alkhateeb, Ph.D. †

Abstract

This study was designed to identify and examine differences between expert and novice students' understandings of the area and perimeter of a rectangle. One hundred eight students (M = 11.3 yr., SD = 0.48, range = 11 to 13 yr.) were asked to respond to an essay question that asked them to explain the area and perimeter of a rectangle mentioning any ideas, concepts, or formulas they think are necessary or helpful. Students' responses were rank-ordered on a rubric, and were analyzed using qualitative methodology to search for characteristics that discriminated between expert and novice students. Results suggest differences in the ways expert and novice students understand the concept of the area and perimeter. Specifically, evidence is presented indicating that using formulas, examples, and drawing strategies can play a significant role in expert responses. Recommendations include that the strategies observed among the experts be incorporated into teaching mathematics in general, but especially at Grade 5, 6, and 7, where standards related to area and perimeter of a rectangle are listed. The identified strategies provide teachers with valuable clues concerning instruction and assessment strategies for mathematics students, especially when teaching the concepts of area and perimeter of a rectangle.

Background

The purpose of this research study is to assess students' conceptual understanding of area and perimeter among 6^{th} grade students through essay writing. Specifically, the following research question is addressed: How do novice and expert students of 6^{th} grade mathematics exhibit differences in the way they explain the concepts of area and perimeter of a rectangle?

Because the measurement topic is an important component of K-12 mathematics curricula, students are expected to understand it as an integral part of their mathematical literacy. A standard involving understanding the concepts of area and perimeter of squares and rectangles is first introduced in 3^{rd} grade, according to the Qatar Mathematics Curriculum Standards (Supreme Education Council, 2004). At this level, students are expected to know that perimeter is the distance measured around the boundary of a figure. They do simple problems that involve finding the perimeter and area of squares and rectangles. An example is: "*Here is a centimeter square grid. On the grid, draw a rectangle with a perimeter of 10 cm*" (Supreme Education Council, 2004, *p*. 88). The concept is revisited again in 4^{th} grade, where students find the area and perimeter of simple shapes with problems such as, "*A thin wire 20 centimeters long is formed into a rectangle. The width of the rectangle is 4 centimeters. What is its* *length?*" (Supreme Education Council, 2004, *p*. 102). In 5th grade, students solve simple problems that involve finding the areas and perimeters of shapes related to rectangles and squares, and the volumes of cuboids, for example, "*The area of a square is 64 cm^2. What is the length of its perimeter?*" (Supreme Education Council, 2004, *p*. 116). The study of area and perimeter measurement is an important part of the 6th grade mathematics curriculum for three important reasons: firstly, because of the wide variety of everyday applications of area and perimeter concepts in activities such as painting, tiling, distance and indeed any task which involves dealing with two dimensional surface; secondly, because area and perimeter concepts are often used in textbooks and by teachers to introduce many other mathematical ideas; and thirdly, this is a critical time in student's education of mathematics in Qatar as it prepares them for middle school mathematics.

Writing is an important part of learning mathematics. Across all grade levels, the Principles and Standards for School Mathematics (NCTM, 2000) recommended writing as a way of communicating mathematics and strengthening students' mathematical thinking, because it requires them to reflect on their work and to clarify their thoughts about the ideas and concepts. Emphasis is that students should have frequent opportunities to express their thinking in writing, because the ability to make mathematical thinking observable helps them clarify their thoughts. It also enables them to use mathematical language that is more precise to express their ideas and deepen their understanding of mathematics in order to become better mathematical thinkers.

Research also indicates that writing is a valuable element of mathematics learning (Beidleman, Jones, & Wells, 1995; and Meel, 1999). Mathematics educators recognize the use of reading and writing in communicating and learning mathematics. Bosse and Faulconer (2008) outlined procedures that can be employed in mathematics assessment to create experiences that promote reading and writing as tools for expressing mathematics understanding. Donna Alvermann (2002) urges all teachers, irrespective of their content area expertise, to encourage students to read and write in a variety of ways. Pugalee (2005), who researched the relationship between language and mathematics learning, proclaims that writing supports mathematical reasoning and problem solving and aids students internalize the characteristics of effective communication. He recommends that teachers read student writing for evidence of logical conclusions, justification of answers and processes, and the use of facts to explain their thinking. Learning to write about mathematics using a mathematics rubric helped improve third graders' competencies when explaining solution strategies in writing (Parker & Breyfogle, 2011).

Educators and researches often notice that students have difficulty with area measurement. For instance, students often confuse the area of a rectangle with its perimeter. A review of the literature (e.g., Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998; Doig, Cheeseman, & Lindsey,1995;

Kamii & Kysh, 2006; Lehrer & Chazan 1998; Outhred & Mitchelmore, 2000; Reynolds & Wheatley, 1996) has indicated that student difficulty with the area tasks is often due to their lack of conceptual understanding of area as a quantitative attribute. Other studies also have assessed the confusion between perimeter and area (e.g., Lehrer, Jenkins & Osana, 1998); Outhred and Mitchelmore (2000) stressed on importance of conceptual understanding of length measurement to understand the area measurement. Meaning, if teachers could identify the growth of students' conceptual understanding of length and area measurement, they would be able to improve the teaching of these topics. For example, Kidman (1999) found that students often confuse area and perimeter, and tend to use addition to calculate areas when multiplication strategies would be more beneficial; Dickson (1989) noted that students have a strong inclination to employ the rectangular area formula ($Area = length \times width$) in all contexts, regardless of the shape; and Baturo and Nason (1996) revealed that such misconceptions are often deeply held and can continue to a later age. Curry, Mitchelmore and Outhred (2006) examined development of kids' understanding of length, area and volume measurement in grades 1 through 4. Students in their study and based on incorrect reasons often rejected the use of different sized units for area or volume but did not see a problem with using different sized units for length. Curry, Mitchelmore and Outhred note that "young students appear to have a much poorer understanding of the need for identical units that leave no gaps than teachers often assume, and they may indeed have no clear concept of what they are measuring" (p. 383).

Research on cognitive skills shows that there are qualitative discrepancies in the knowledge and thinking of experts and novices. For example, novices, unlike experts, underuse visual representations during problem solving (Kozma, 2003). Studies of both experts and novices show that the use of visual problem representations facilitates thinking and problem solving performance (Moreno, Ozogul, Reisslein, 2011).

If knowledge is considered to be the making of connections or associations between small elements of experience, a difference emerges between an expert and a novice at mathematics. The expert has a larger and richer corpus of associations between the basic links of mathematics and the bigger patterns and processes of mathematics (Hughes, Desforges, & Mitchell, 2000). Studies of expertise in fields such as medical diagnosing (Norman, 2005) show that experts not only have more knowledge about their field of expertise, but also that their relevant knowledge is organized in long-term memory in such ways that make it more accessible when it is needed.

The present study seeks a different, but complementary, emphasis on expert/novice distinctions. First, the largely unexplored method of essay questions is used to obtain the subjects' responses. Secondly, the study focuses on the concepts of area and perimeter of 6th grade students in Qatar. Very little is known about how experts learn these concepts, and studies of this kind are rare in the region. With the current extensive call for reform in mathematics education in Qatar, it is imperative that the difficulties facing elementary students in mathematics be examined. Moreover, little work appears in the literature on expert and novice in elementary mathematics; it is largely studied in domains such as university courses, teacher preparation, and professional development.

Purpose

Using an expert/novice contrast, this study assesses students' understanding of the concepts of area and perimeter of rectangles, and outlines how these concepts are understood by both expert and novice students. The results provide a dual opportunity. One is to deepen the understanding associated with elementary students' learning of area and perimeter; the second is to contribute to the research literature regarding mathematics education in Qatar, because it represents the debut of expert/novice contrast of elementary mathematics in Qatar.

Methodology

Participants were 108 students from two elementary schools (one boys school and one girls school) in Doha, Qatar. The sample included 51 (47%) boys and 57 (53%) girls. Their ages ranged from 11 to 13 years, with a mean age of 11.3 years (*SD* = 0.48 years). Among the sample, 105 (97%) were Qatari Nationals and the remaining three (3%) were of Egyptian and Yemeni descent.

In the middle of the academic year, students were given an essay question in which they were to write about their conceptual understandings of the area and perimeter of a rectangle. They were to address what the concepts mean in a mathematics context, and to mention any necessary and helpful ideas or concepts in their response. The essay question stated, "Explain, in details, the concept of the area and perimeter of a rectangle. Cite relevant ideas, concepts, and formulas to support your explanation."

The researcher, using a carefully developed and validated performance assessment rubric graded students' responses to the research question. Student essays were rank-ordered according to total possible score of 16. For analysis, the top 25% (27 essays) were identified and referred to as the "experts," and the bottom 25% (27 essays) were labeled as the "novices." Of the 27 expert students, 10 were boys and 17 were girls. In the novice group, 15 students were boys and 12 were girls. The teaching style in these mathematics classes was mainly traditional—consisting of large-group lectures and, sometimes, small-group problem-solving discussions.

Data collection also included demographic information, such as the student's age, grade, gender, nationality, and midyear achievement grade in mathematics. Students' midyear grades were provided by the sixth grade teacher and recorded on the hard copy of the research essay response. Students did not provide names or identification numbers since they were not required for matching purposes.

Qualitative content analysis was performed on the collected data, for which frequency analysis was used. Statements that could possibly have bearings upon conceptual understanding were given attention. Data were assigned to categories and the frequencies and percentages were calculated. The findings were transformed by making connections across categories. In the transformation process, loadings associated with area and perimeter were interpreted. Feedback to participants was not provided.

Results

An expert/novice qualitative examination of students' responses resulted in the identification of four categories, differences that distinguished expert students from novice students. The terms "expert" and "novice" in this study refer only to their skill in approaching this specific problem, and not their overall skill in mathematical writing (a more general issue). The overall students' midyear mathematics achievement ($M = 69.4$, $SD = 17.7$, $N = 108$) was moderate. The midyear mathematics achievement for boys ($M = 68.6$, $SD = 17.5$, $N = 51$) was comparable to girls ($M = 70.1$, $SD = 18.1$, $N = 57$). The difference between the midyear mathematics achievement of the novices ($M = 60.5$, $SD = 15.1$, $N = 27$) and experts ($M = 79.8$, $SD = 14.0$, $N = 27$) was significant, $t_{52} = -4.874$, $p < .001$, $d = 1.35$, a large effect according to the criteria proposed by Cohen and Cohen (1983). The categorization of responses indicated that expert students were better able to use formulas, definitions, drawings, examples, and organization of their work than were the novice students. These results are in line with other findings on expert/novice distinction (e.g., Hardin, 2002; Moreno, Ozogul, & Reisslein, 2011).

The following are four (4) major findings from the study.

1. Expert students began their essay with a clear definition or formula of the area and perimeter, or they were more likely to use the area and perimeter definition or formula mentally as part of their answer—showing a deep understanding. Novice students were less likely to try a definition or formula, and were more likely to miss an essential part of the definition or formula—showing a more superficial understanding.

   Student A, identified as an expert, began the essay with "to find the area of the rectangle, you have to multiply the width by the length; and to find the perimeter of the rectangle; you have to find the sum of the measure of all of its sides." Student *B* wrote "area = length × width, and perimeter = 2 × (length + width)." Student *C* provided a rectangle of 3 *cm* by 4 *cm* and dissected it to make 12 squares (each is 1 *cm* by 1 *cm*) and wrote "area = sum of the areas of the squares (or number of squares inside the shape)." Of the 27 expert students, 15 (56%) of them followed this example for the area with slight variations; 21 (78%) of them followed this example for the perimeter with slight variations.

Responses categorized as novice tended not to start so strongly—giving the (incorrect) definition or formula or not providing a definition or formula at all. For example, Student Y left the area question mostly blank and without either a definition or formula. But for the perimeter, Student Y indicated, "You multiply the lengths of the four sides of the rectangle."

Others began with an incorrect or vague definition or formula of the area and perimeter, as in the case of Student Z, who wrote, "multiply the sides" for both concepts. Student X tried to define the rectangle instead of touching on the area and/or perimeter, and wrote for the area, "it has 4 sides, it has equal sides, the angle is 90°, it has four corners," and "the rectangle is a type of a square but not a square." Student W stated for the perimeter, "we add together"; no application or example was provided. Student V wrote for the perimeter question, "sum the sides"; no other information was provided.

A high proportion of responses for the area concept (26%), and for the perimeter concept (19%), however, started with putting in numbers and multiplying and/or adding them to find the area and/or perimeter, but did not follow the correct procedure. For example, Student U added the length of all sides to find the area, and multiplied all sides to find the perimeter.

2. Experts presented more ways to explain the concepts of area and perimeter than novices. Expert students had a higher tendency to support their answers with drawings, providing specific examples of rectangles that included side lengths and units than novice students. Novices did not provide such drawings or specific examples of rectangles.

Perhaps the most basic way of understanding the concepts of area and/or perimeter is through visualization. The use of graphics for clarification was more common among the expert students (all of them drew rectangles). For example, Student A, an expert student, provided a rectangle labeled with a specific length for each side including the unit (e.g., 6 *cm*, 3 *cm*, 6 *cm*, and 3 *cm*). 25 (93%) of the 27 expert students, including Student A, followed this example with slight variations. An example of a variation was provided by Student D, who drew a rectangle labeled with numbers correctly, but did not assign a correct unit; instead, providing 2 cm^2, 4 cm^2, 2 cm^2, 4 cm^2 as the side lengths. The student, however, provided the correct unit for the area, stating it as "A = 4 × 2 = 8 cm^2." Aside from mislabeling the unit of the side length, it was a strong example.

The generally lower level of understanding prevented novice students from giving specific examples of area and/or perimeter or from using a correct shape. Of the 27 novice students, 13 (48%) used drawings in their essay in some way. For example, Student Y (a novice) provided a triangle instead of a rectangle for the area question; it was not labeled with any numbers or units except with the word "triangle." For the perimeter, Student Y provided a rectangle labeled with 3, 3, 3, and 9 (A unit was not given.), and these numbers were placed in the inside corners of the rectangle where angles are usually labeled.

Others, such as Student U, drew a rectangle labeled with 12, 4, 12, and 4 (A unit was not given.) and dissected it into 48 square units but did not use these square units to calculate the total area. For the perimeter, Student U drew the rectangle as an oval shape labeled with four dimensions 2, 5, 2, and 5 (A unit was not given).

Student T drew a rectangle for the area and a triangle for the perimeter labeled with only the words "rectangle" and "triangle," respectively. This was the entire response provided in Student T's essay.

Student S, another novice, provided two shapes for the area, a rectangle labeled with 2 *cm*, 4 *cm*, 2 *cm*, 4 *cm;* and an isosceles triangle labeled with 4 *cm*, 5 *cm*, and 5 *cm*.

Student J drew an equilateral triangle for the area question. It was labeled with "1" on each side (A unit was not given.) Unfortunately, these examples provided little or no insight on what the concepts of area and/or perimeter actually mean.

3. Expert students' responses more often gave meaningful and correctly-solved examples referring to the area as $width \times length$ and perimeter as $twice\ the\ length + twice\ the\ width$. Novice students only said that the area and perimeter has something to do with the sides.

Expert students have a stronger grasp of the concept of area and perimeter and exactly what they mean. Student A stated that "area = 6 *cm* × 3 *cm* = 18 cm^2" and "perimeter = 6 *cm* + 3 *cm* + 6 *cm* + 3 *cm* = 18 *cm*." Student B stated that area = length × width = 10 × 5 = 50 (A unit was not given.); and stated that perimeter = 2 × (5 + 10) = 30 (A unit was not given.)"

While the novice students might have had some idea that the concepts of area and/or perimeter and the side lengths were strongly connected, very few of them followed the right procedure for calculating the area and/or perimeter, and those who did calculate, missed details – some minor; some major. Student Y, a novice, did not provide any calculations for the area but stated, "…perimeter = 3 × 3 × 3 × 9 = 118." Student U stated, "…area = 12 + 12 + 4 + 4 = 32" and stated, "…perimeter = 5 × 5 × 2 × 2 = 100." Student S who provided only a labeled rectangle and a labeled isosceles triangle for the area question with no further details for this part, calculated the perimeter for each shape stating that the perimeter of rectangle = 4 + 4 + 2 + 2 = 8 *cm,* and that the perimeter of triangle = 5 + 5 + 4 = 14 *cm*. Student J's equilateral triangle for the area question stated, "…area = 1 × 1 × 1 = 1 cm."

4. Expert students were better prepared than novices at choosing and diversifying strategies. Expert students were more organized and presented diagrams, calculations, and results more clearly and more precisely than novice students. Their work was not well organized and important data was difficult to locate.

   Some students provided rectangles similar in shape to a variety of geometric figures, such as a trapezoid, an octagon, or an oval shape. Student R, a novice student, provided an imprecise rectangle (more trapezoidal than a rectangular) labeled with the measures 3 *cm*, 4 *cm*, 3 *cm*, and 4 *cm*. These measures were placed in the interior of the drawn shape instead of the exterior, which is the conventional way. The area was gives as, "…area 3 × 3 × 4 × 4 =132."

   Another novice student, Student Q, provided a final answer for the area that was difficult to understand – including a rectangle (not well drawn) with the dimensions 10, 15, 10, 15 (A unit was not given.); yet the student wrote this for the area: "A = 10 × 15 × 10 × 15 = 50." No information about the perimeter was provided from Student Q. While some novice students provided reasonable lengths, their calculations did not convey a correct understanding of the area.

Discussion

In describing expertise, Klein and Hoffman (1993) state, "…novices see only what is there, while experts can see what is not there" (p.203). Ericsson and Lehman (in van Gog, Ericsson, Rikers, and Paas, 2005) describe experts as showing "consistently superior performance on a specified set of representative tasks for a domain (p. 277)." An expert is expected to have a high degree of proficiency, skill, and knowledge in a particular subject.

Research literature reports that experts and novices from different fields perform differently (Benner, 1984; Swanson, O'Connor, & Cooney, 1990; Chae, Kim, & Glass, 2005; Voss, Kunter, & Baumert, 2011; Rey & Buchwald, 2011). Results from this study concur with those from previous research.

Moving from novice to expert involves considerably more than developing a set of generic skills and strategies (Harper, 2007). In analyzing students' writing of the concepts of area and perimeter, the results of this study highlight that the expert and novice students exhibit differences in the way they perceive these two concepts.

Novice students have difficulty with the definition and/or formula of the area and perimeter. They do not relate the area and/or perimeter to specific examples as easily as expert students. The experts are better prepared than the novices to answer the essay question by using specific examples. For many students, applications of specific examples represent the most useful facets of understanding these concepts. Overall, experts seem to be relatively consistent in the examples/solutions and explanations they write. Additionally, experts are more organized; they present their writings clearly and precisely. These results are also consistent with earlier findings of a gap between experts and novices (Norman, 2005).

Visual demonstrations seem to be of particular benefit for experts. By concentrating on these differences, additional progress might be made to reduce the gap between the experts and the novices by helping the novice student see things the way the expert does. Interestingly, findings from statistics (Quilici & Mayer, 1996) show that novices can be taught to represent problems on a more expert level through instruction and practice.

Whereas the research regarding expert/novice differences within mathematics has merit, there are several important limitations in this study that must be acknowledged. First, essay responses are the sole source of the assessment data provided for analyses and comparisons in the study. Second, the study is limited to two topics in mathematics – area and perimeter of a rectangle. Despite these limitations, the study provides a rich analysis through the comparison of responses demonstrating different levels of student understanding. This subsequently offers insights into how students' conceptual understanding can influence the way these concepts should be taught. Moreover, this study also informs the structure and content of teachers' professional development so that the evolution from novice to expert can advance at a more rapid pace.

In conclusion, the present study extends our understanding of student learning and provides information that is important to educators who seek ways to improve students' conceptual understanding through writing-to-learn mathematics. This study provides a context that suggests that the observed expert/novice distinctions in learning mathematics are worthy of further investigation in other mathematics topics and at different grade levels to determine the representativeness and generalizability of these results. Finally, the nature and sequence of student learning experiences need to be investigated before offering recommendations that enable novices to evolve toward expertise. Given that expert and novice students show understanding differently, additional research is needed to observe whether novice students can be taught the skills of experts and how it can be accomplished. Finally, more research is needed to understand when and how students' views regarding mathematics concepts shift, and what experiences help students move from being novices to becoming expert students.

† *Haitham M. Alkhateeb, Ph.D.,* University of Baltimore, Baltimore, USA

References

Alvermann, D. E. (2002). Effective literacy instruction for adolescents. *Journal of Literacy Research*, 34, 189 – 208.

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' spatial structuring of 2D arrays of squares. *Journal for Research in Mathematics Education, 29*(5), 503-532.

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics*, 31, 235–268.

Beidleman, J., Jones, D., & Wells, P. (1995). Increasing students' conceptual understanding of first semester calculus through writing. *Primus (Problems, Resources, and Issues in Mathematics Undergraduate Studies)*, 5(4), 297-3 16.
Benner, P. E. (1984). *From novice to expert: Excellence and power in clinical nursing practice*. Menlo Park, CA: Addison-Wesley.

Bosse, M. J., & Faulconer, J. (2008). Learning and Assessing Mathematics through Reading and Writing. *School Science and Mathematics*, 108, 8 - 19.

Chae, H. M., Kim, J. H., & Glass, M. (April 2005). Effective Behaviors in a Comparison Between Novice and Expert Algebra Tutors. *Proceedings. Sixteenth Midwest AI and Cognitive Science Conference. MAICS2005*, Dayton, pp. 25-30.

Cohen, J., & Cohen, P. (1983). *Applied multiple regression/correlation analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Erlbaum.

Curry, M., Mitchelmore, M, & Outhred, L. (2006). Development of children's understanding of length, area, and volume principles. In J. Novotná, H.

Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 377–384, Prague: PME.

Doig, B., Cheeseman, J., & Lindsey, J. (1995). *The medium is the message: Measuring area with different media.* Paper presented at the Galtha: Eighteenth Annual Conference of the Mathematics Education Research Group of Australasia, Merga.

Hardin, L. E. (2002). Problem Solving Concepts and Theories. *Journal of Veterinary Medical Education*, 30(3), 227 – 230.

Harper, K. A. (October 2007). *Making Problem Solving a Priority.* Presented at *the 32nd Annual Conference of the Professional Organization and Development (POD) Network, Pittsburgh, Pennsylvania, U.S.A.*

Hughes, M., Desforges, C., & Mitchell, C. (2000). *Numeracy and beyond: applying mathematics in the primary school.* Open University Press, Buckingham, UK.

Kamii, C., & Kysh, J. (2006). The difficulty of "Length x width": Is a square the unit of measurement? *The Journal of Mathematical Behavior, 25*(2), 105-115.

Kidman, G. C. (1999). Grade 4, 6 and 8 Students' Strategies in Area Measurement. In J. M. Truran & K. M. Truran (Eds), Making the Difference (Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia, pp. 298–305). MERGA, Adelaide.

Klein, G. A., and Hoffman, R. R. (1993). Seeing the invisible: Perceptual-Cognitive aspects of expertise. In M. Rabinowitz (Ed.), *Cognitive Science Foundations of Instruction* (*pp*. 203-226). Hillsdale NJ: Erlbaum.

Kozma, R. (2003). The material features of multiple representations and their cognitive and social affordances for science understanding. *Learning and Instruction, 13, 205–226. doi:10.1016/S0959-4752(02)00021-X*

Lehrer, R. & Chazan, D. (Eds.). (1998) *Designing learning environments for developing understanding of geometry and space*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 137-167). Mahwah, NJ: Lawrence Erlbaum.

Meel, D. E. (1999). Journal writing: Enlivening elementary linear algebra. *Primus (Problems, Resources, and Issues in Mathematics Undergraduate Studies)*, 9, 205-225.

Moreno, R., Ozogul, G., Reisslein, M. (2011). Teaching with concrete and abstract visual representations: effects on students' problem solving, problem representations, and learning perceptions. *Journal of Educational Psychology*, 103, 32 – 47.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Norman, G. (2005). From theory to application and back again: Implications of research on medical expertise for psychological theory. *Canadian Journal of Experimental Psychology*, 59, 35–40.

Outhred, L. N., & Mitchelmore, M. C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education*, 31(2), 144167.

Parker, R, Breyfogle, M. L. (2011). Learning to write about mathematics. *Teaching Children Mathematics*, 18, 90 – 99.

Pugalee, D. K. (2005). *Writing to develop mathematical understanding*. Norwood, MA: Christopher-Gordon.

Quilici, J. L., & Mayer, R. E. (1996). Role of examples in how students learn to categorize statistics word problems. *Journal of Educational Psychology, 88*, 144 - 161.

Rey, G. D., & Buchwald, F. (2011). The expertise reversal effect: cognitive load and motivational explanations. *Journal of Experimental Psychology: Applied*, 17, 33 – 48.

Reynolds, A., & Wheatley, G. H. (1996). Elementary students' construction and coordination of units in an area setting. *Journal for Research in Mathematics Education, 27*(5), 564-581.

Swanson, H. L., O'Connor, J. E., & Cooney, J. B. (1990). An information processing analysis of expert and novice teachers' problem solving. *American Educational Research Journal, 27*,
533–556.

Supreme Education Council. (2004). *Curriculum Standards for the State of Qatar: Mathematics (Grades K -12)*. Doha, State of Qatar: Author.

van Gog, T., Ericsson, K. A., Rikers, R. M. J. P., & Paas, F. (2005). Instructional Design for Advanced Learners: Establishing Connections between the Theoretical Frameworks of Cognitive Load and Deliberate Practice. *Educational Technology Research & Development*, *53*(3), 73-82.

Voss, T, Kunter, M., & Baumert, J. (2011). Assessing teacher candidates' general pedagogical/psychological knowledge: test construction and validation. *Journal of Educational Psychology*, 103, 952 – 969.