Induced Compatibility relation and its Application to Knowledge Database Implementation

D. Singh, Ph.D. †
T.O. William-west ‡

Abstract

In this paper, we crisply introduce *induced compatibility* relation defined on a finite set and thereby, discuss its application to knowledge database implementation in an *expert system*. Essentially, it is shown that an *intelligent machine* may implement a task with the help of a compatibility relation defined between the task and its knowledge database.
Keywords: compatibility relation, compatibility classes, neural network system

Introduction

Over the last few decades, several applications of compatibility relation have emerged, for example, the problem addressed to minimizing the states of an *incompletely specified sequential machines* [4, 5], finding all the *maximal* subgraphs of a graph G, etc, have been efficiently solved by exploiting the notion of *maximal compatibility classes* of a finite set endowed with a compatibility relation.
The algorithms for computin *maximal compatibility* classes have been studied in [2, 3, 5] and recently in [6], for example, it has been applied to network segmentation and decentralization, in the recent years, particularly, to control congested network [6]. In fact, the notion of compatibility relation has gone far beyond its ordinary linguistic connotation and mathematical characterization [3]. From a theoretical point of view, more often than not, elements of a given compatibility class are required to be pairwise compatible. However, from application view point, particularly, in Neural Network system, a set of neurons may be involved in a *process*, even when they are not all pairwise compatible (such as in *feedforward* network system). This paper purposes to discuss such a compatibility relation, hence forth called *induced compatibility* relation.

2. Induced Compatibility Relation

In most of the related literatures, elements of a compatibility class are assumed to be pairwise compatible. In this paper, we wish to modify the said restriction.

2.1 Compatibility Relation

A relation R on a set S is said to be a *compatibility* relation, sometimes denoted $\approx$, if it is reflexive and symmetric. Obviously, all equivalence relations are compatibility relations.

Let S be a finite set and $\approx$ a compatibility relation on S. A subset $M \subseteq S$ is called a compatibility class, if any element of M is related to its every other element. In addition, M is said to be a *maximal* compatibility class if no element of $S - M$ is compatible to all other elements of M. In [6], some properties of maximal compatibility classes have been investigated and some results obtained.

2.2 Induced Compatibility Relation

We introduce the operation $\oplus$ to denote the *induced* compatibility relation. In fact, for simplicity, the familiar symbol $+$ for addition can be used if there is no confusion. However, in such case, an equation is flagged with the symbol (I) at the end to indicate its induced sense of use. As we shall use $\oplus$, no use of (I) will be used unless it becomes useful for clarity.

Let S be a finite set, for $x, y \in S$, define $\oplus$ on S by $x \oplus \bigcup_{i=1}^{k} y_i$, where $\bigcup_{i=1}^{k} y_i = \{y_1, y_2, y_3, \dots y_k\}$ are pairwise compatible. If there exist $y \in \bigcup_{i=1}^{k} y_i$ such that $x \approx y$. Then we say x is inducible to $\bigcup_{i=1}^{k} y_i$, and call the relation *induced* compatibility relation. It is not difficult to see that every induced compatibility relation can be made into a compatibility relation.

The idea behind the induced compatibility relation besides imposing a compatibility relation between an element and a compatibility class, is to relax the *pairwise compatibility* criteria in order to obtain a suitable operation which is admissible in neural network settings, without altering the compatibility relation that may exist between neurons of the network. Also, when pairwise compatibility between elements of set S is given a nominal scale, the induced compatibility relation gives a proximity relation (in compatibility sense) between members of S and its compatibility classes. This is readily seen by setting $\oplus = d\left(x, \bigcup_{i=1}^{k} y_i\right)$, where d is a distance function. Therefore, the extent to which we can induce a compatibility relation between members of a set can be *estimated*.

Example

Let $S = \{x, y_1, y_2, y_3, \dots y_n\}$ such that $x \approx y_1$ and $C = \{y_1, y_2, y_3, \dots y_4\}$ a compatibility class (pairwise compatible) with a nominal scale of 0.4. Then the induced compatibility relation $\oplus$ is defined by $x \oplus \bigcup_{i=1}^{k} y_i$ and define the following graph, if $\oplus = d\left(x, \bigcup_{i=1}^{k} y_i\right)$, where the quadrilateral is the compatibility class C and the line joining the quadrilateral to x (external point) is a compatibility class $C_1 = \{x, y_1\}$.

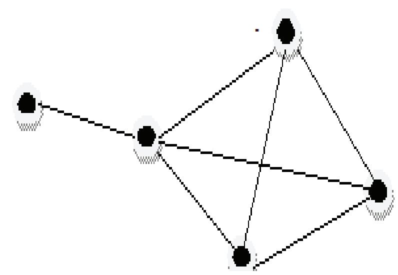

Fig. 1: Simplified graph of the induced compatibility relation

Theorem 1

Let S be a finite set endowed with a compatibility relation. Then every element of S is inducible to some compatibility class of S.

Proof: Let x be an arbitrary element of S. Then either x is not compatible to any element of S (in such case, trivially, $x \oplus \cup\, x = x$) or x is compatible to some (but not all) elements of S. Or x is compatible to all elements of S. In any case, the result holds.

Algebra of the Induced Compatibility Relation (ICR)

Let $\oplus$ and $\approx$ denote the induced compatibility relation and the compatibility relation on finite set S respectively. For $x, y_j, y_k \in S$. The following result holds.

i. $y_k \approx x \oplus \bigcup_{j=1}^{k} y_j = x \oplus \bigcup_{j=1}^{k+1} y_j \,, if\ y_k \approx y_j$.

ii. $y_k \oplus x = x \oplus y_k$.

iii. $y_k \oplus x = x \approx y_k$.

The following theorem will bring home the algebra of this relation.

Corollary 2

Let S be a finite set endowed with a compatibility relation. Let $\oplus$ be an induced compatibility relation on S. Then $\oplus \bigcup_{i=1}^{k} y_i = \ y_k \oplus \bigcup_{i=1}^{k-1} x, y_i$, if $y_k \approx y_i$, for all $x, y_{i=1,2,\ldots,k-1} \in S$.

Proof: Let S be as in the statement of theorem 3. From R.H.S., $y_k \oplus \bigcup_{i=1}^{k-1} x, y_i$ can be written as: $y_k \oplus \{x, y_1, y_2, \ldots, y_{k-1}\}$

$= \ y_k \oplus (\{x\} \cup \{y_1, y_2, \ldots, y_{k-1}\})$

$= (\{x\} \oplus y_k) \cup (y_k \oplus \{y_1, y_2, \ldots, y_{k-1}\})$

$= (\{x\} \oplus y_k) \cup \{y_1, y_2, \dots, y_k\}$
$= \{x, y_k\} \cup \{y_1, y_2, \dots, y_k\}$
$= x \oplus \bigcup_{i=1}^{k} y_i$

Theorem 3 (Fundamental theorem of ICR)

Let S be a finite set endowed with a compatibility relation and C_1 *and* C_2 be distinct compatibility classes of S. If $x \oplus C_1$ and $x \oplus C_2$, for $x \in S$. Then $x \oplus C_1 \cup C_2$. In general, if $x \oplus (C_1, C_2, \dots, C_n)$. Then $x \oplus \bigcup_{i=1}^{n} C_i$.
Proof: Easy!

Remark 1

For the case of intersection, the statement of theorem 3 may not hold. That is, if $x \oplus C_1$ and $x \oplus C_2$, it may not be the case that $x \oplus C_1 \cap C_2$.

Definition 1

Let C_r *and* C_s be compatibility classes of finite set S, for $r \neq s$. We say that C_r is inducible to C_s, i.e., $C_r \oplus C_s$ if for some $y_i \in C_r$ we have $y_j \in C_s$ such that $y_i \approx y_j$, for some i, j. This type of compatibility relation is a level two relation.
By definition 1, it can be verified that $x \oplus (C_r \oplus C_s) = (x \oplus C_r) \oplus C_s$, for $x \in S$.
In the system $(S, \oplus, \approx)$, $\approx$ is distributive over $\oplus$. That is, for $x, y, z \in S: x \approx (y \oplus z) = (x \approx y) \oplus (x \approx z)$. However, it is not always true that $\oplus$ distributes over $\approx$.

2.3 Suggested Framework

From application point of view, the assumption of this work is that; the intelligent system under consideration is equipped with some not necessarily disjoint subclasses of knowledge database which has some relation with the task to be implemented. Moreso, we assume that the knowledge database can be represented as: agent A, and patient P, which is connected by the underlying compatibility relation. In our framework, we assume that there exist a pattern which retrieves the agent whenever the patient is known and vice versa. Consequently, we can represent knowledge by accounting for the compatibility relation that exist between the agent and patient. For example, the knowledge *Singh is a senior staff* can be accounted for by an expert machine by investigating the compatibility relation that exist between the database

containing the names of all staff, including *Singh* and the one containing all *senior staff*. In this respect, *Singh* is the agent of the relation and *senior staff* is the patient.
It should be emphasized that in most cases, an agent of the relation could be view as a patient as well. This is guaranteed by the symmetric property of the relation. More often than not, if the compatibility relation exist (and is known) between the agent and patient, if the agent is also known, we may ask about the patient. In such case, a possible query might be formed thus; what category of staff is *Singh*? For such a query, it is observed that a system which accounts for such relationship to represent knowledge, is capable of remembering relationship of objects and may implement this knowledge database efficiently if the underlying relation is a compatibility relation. More explicitly, we may consider an intelligent machine (a robot) equipped with knowledge database. If the machine has to implement or process a set of task which is connected to some pieces of data in its memory. What might be appreciated from this is elaborated as follows:
Let the task t to be implemented have some connection with some data d_i in the knowledge database of the machine. Then $t \oplus \bigcup_{i=1}^{k} d_i$, where d_i is the resources required to implement the task.
It is important to note that t is compatible to some or all $d_i's$. Therefore, when task t is to be implemented, the machine pops out all the resources frames d_i related to $d_k \approx t, for\ some\ k$. If t does not directly relate to $d_1, d_2, \dots, d_{k-1}$, additional memory capacity is consumed, but there is a compensation for this memory consumption. The compensation being that, through d_k additional resources (which may serve as possible alternatives) is supplied to t to meet its demand. On the other hand, if corresponding to task t is some resources $d_1, d_2, \dots, d_k$, the possibility that the machine will undergo a decision problem (i.e., which of the $d_i's$ is to be utilized) is overruled. Since by symmetry, each d_i is as good as the other provided $\{d_1, d_2, \dots, d_k\}$ is a compatibility class.
Schematically, this can be viewed as a frame (*neuron*) connecting other frames by means of an induced compatibility relation. The diagram below describes the idea.

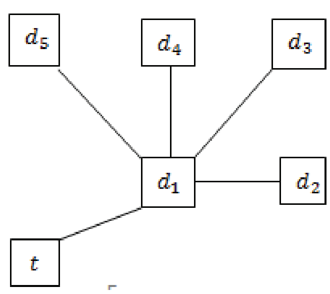

Fig. 1: Diagram representing task t compatible to only one member $d_1$of compatibility classes $\{d_1, d_2\}$, $\{d_1, d_3\}$, $\{d_1, d_4\}$, $\{d_1, d_5\}$

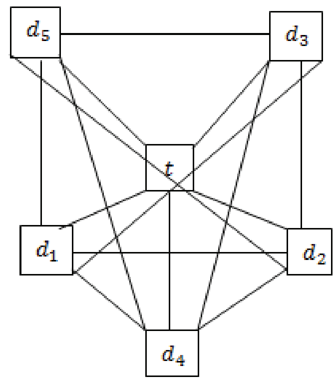

Fig. 2: Diagram representing task t compatible to all members of compatibility class $\{d_1, d_2, \dots, d_5\}$.

It is observed that a set of tasks $T = \{t_1, t_2, \dots, t_n\}$ may be inducible to some or all of the data $D = \{d_i: 1 \leq i \leq k\}$. This type of compatibility relation is a level 2 relation as in definition 1. On the other hand, a piece of data may serve as resource to a set of tasks T. Therefore, it might be difficult to assign precise nominal values to direct relation between these neurons or frames. However, since neural networks are capable of *learning* [1], this difficulty can be overcome.

Concluding Remarks

In this paper, we have presented the notion of *induced compatibility relation*, which, to our knowledge, has not been implicitly explicated so far. Definitionally, the element which is inducible to a compatibility class, together with the class itself, gives rise to an induced compatibility class. The main theme of the paper is to show that the notion of compatibility relation may be employed to efficiently implement knowledge database tasks. However, this notion, when being applied may cause the expert system to generate all those frames which are compatible (in induced sense) to the task to be implemented. Hence each frame must be checked on how appropriately it resolves the task. There is, however, a payoff for this extra work of generating and checking, namely, this notion is simple to implement. For this reason and, with the advances in very large scale integration (VLSI) technology, easy implementation may be available which would provide a significant advantage.

† *D. Singh Ph.D.*, Ahmadu Bello University, Nigeria
‡ *T.O. William-west*, Ahmadu Bello University, Nigeria

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