An Application of Fuzzy Sets for Studying the Influence of Computational Thinking in Learning Mathematics

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Abstract

Computational thinking is a term recently introduced to describe a set of thinking skills that are integral for systems' analysis, for solving complex problems and for the understanding of the human behavior, by drawing on principles fundamental to computer science. In this paper we apply two frequently used in the fuzzy sets theory assessment methods for studying the effects of computational thinking in learning mathematics. The first of these methods concerns the measurement of a fuzzy system's total possibilistic uncertainty, which is connected to the system's mean performance with respect to an activity taking place within it. The second method is an application of the centroid defuzzification technique properly formulated for our purposes, which provides a weighted measure connected to the system's quality performance. Our methods are illustrated by a classroom experiment, the results of which provide a strong indication that the use of computers in teaching mathematics enhances the students' skills for modelling and solving mathematically real world problems. This supports/extends already reported in the literature experimental results (obtained with traditional methods) about the beneficial influence of the use of computers on students' PS skills.

1. Introduction: Critical and computational thinking in problem solving

The importance of Problem Solving (PS) has been realised for such a long time that in a direct or indirect way affects nowadays our daily lives. Volumes of research have been written about PS and attempts have been made by many educationists and psychologists to make it accessible to all in various degrees (e.g. see [12]). Several definitions have been given during the years about PS. Among all these definitions perhaps Martinez's [5] definition carries the modern message about PS: "*PS can be defined simply as the pursuit of a goal when the* *path to that goal is uncertain. In other words, it's what you do when you don't know what you're doing*".

It is a fact that the PS process, not necessarily of mathematical problems only, is a complex situation. Even graduates have nowadays difficulty in solving real life problems. Somehow, they can not apply theory into practice, or theorise/reflect on practice [6]. In fact, it is the human mind at the end that has to be applied in a problematic situation and solve the problem. But human thinking can vary from a very simple and mundane thought to a very sophisticated and complex one. The nature of the problem dictates the level of thinking.

The higher-order thinking, termed as *critical thinking*, involves abstraction, uncertainty, application of multiple criteria, reflection, and self-regulation. The complexity of critical thinking is evident from the fact that there is no definition that is universally accepted. At any case, a great number of critical thinking skills as identified by are agreed upon by many authors. Some of these skills are: analysis and synthesis, making judgements, decision making, reaching to warranted conclusions and generalisations, etc (e.g. see [1]).

With the explosion of information technology and moving away from an industrial society to a knowledge society, the attitude to think critically became a prerequisite (necessary condition) for solving non-routine problems. However, it is not always a sufficient condition too, especially when tackling complicated technological problems of our everyday life, where computers are frequently used as a supporting tool. In this case the need for *computational thinking* (CT) is another prerequisite for PS.

The term CT has been initiated in 1996 by Papert [6], who is widely known as the creator of LOGO, but it was brought to the forefront of the computer society in 2006 by Wing [19], to describe *"a set of thinking skills that are integral for solving complex problems, for systems' analysis and for the understanding of the human behavior, by drawing on the concepts fundamental to computer science"*. The main characteristics of CT include [18]:

- Analyzing and logically organizing data.
- Data modelling, data abstractions and simulations.

- Formulating problems such that computers may assist.
- Identifying, testing, and implementing possible solutions.
- Automating solutions via algorithmic thinking.
- Generalizing and applying this process to other problems.

In general, CT synthesizes critical thinking with the existing knowledge and applies them for PS. Thus, CT is encouraging the application of critical thinking with the help of principles and techniques of the computer science rather, than it is suggesting the solution of problems in the way that computers do. However, the relationship between CT and critical thinking, the two modes of thinking in solving problems, has not been clearly established yet. In an earlier paper [13] we have attempted to shed some light into this relationship. Our conclusions can be summarized with the help of Figure 1, where a 3 - dimensional model for the PS process is presented. According to this model the existing knowledge serves as the connecting tool between critical and computational thinking, while the problem's solution appears to be the "product" of a simultaneous application of the above three components (knowledge, critical and computational thinking) to the PS process. This approach is based on the hypothesis that, when the already existing knowledge is adequate, the necessary for the problem's solution new knowledge is obtained through critical thinking, while CT is applied to design and to obtain the solution.

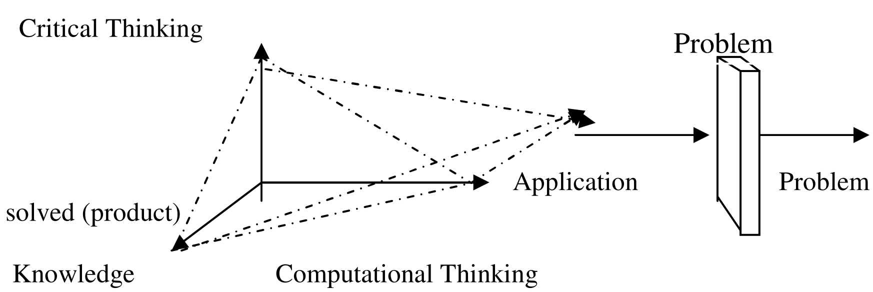

Figure 1: The 3- dimensional model for the PS process

The type of each problem dictates the order of the application of the above three components, which (order) can have in certain, relatively simple, cases the linear form of Figure 2.

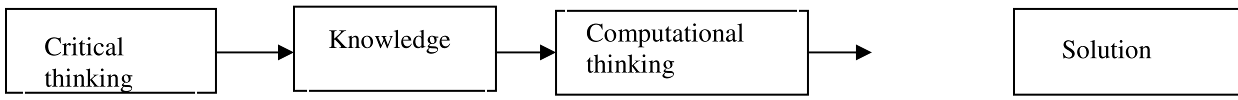

Figure 2: The linear PS model

The above model can be used in formulating the PS process of the complex problems of our everyday life and especially of the composite technological problems.

There are several experimental results reported in the literature, according to which the use of computers as a tool in teaching mathematics enhances the students' PS skills ([4], [13], [17], [20], etc). In using the terminology introduced above, the above results indicate that CT has a beneficial influence on students' PS abilities. All these results have been obtained on comparing the performance of student groups for which computers have been widely used in the teaching process (experimental groups), with respect to the performance of similar groups, where the traditional teaching methods (theory and examples on the board) have applied (control groups).

The assessment methods used in all the above experiments are based on principles of the classical logic (YES-NO). However, fuzzy logic provides a series of more realistic assessment methods, mainly due to its possibility to characterize a situation with multiple values. Our purpose in this paper is to use such kind of assessment methods for studying the influence of CT in learning mathematics. Thus, the rest of the paper is organized as follows: In the next section we present briefly the assessment method connected with the measurement of a fuzzy system's total possibilistic uncertainty. In the third section we present and we properly formulate for our purposes the defuzzification method of the center of gravity (COG), while in the fourth section we illustrate the use of the above two methods in a classroom experiment related to CT. Finally, the last section is devoted to conclusions and a short relative discussion.

For general facts on fuzzy sets and the connected to them uncertainty we refer to the book [2].

2. Measuring a fuzzy system's uncertainty

In measuring a fuzzy system's effectiveness (i.e. the degree of attainment of its targets) with respect to an activity performed within the system (e.g. solving or evaluating the solution of a problem, decision making, etc) it is necessary to use a *defuzzification method* converting the fuzzy data obtained by this activity to a crisp number.

One of the most frequently used such methods is based on the measurement of the system's uncertainty. In fact, from the classical information theory it is well known that the amount of information obtained by an action can be measured by the reduction of uncertainty resulting from this action. Accordingly a system's uncertainty is connected to its capacity in obtaining relevant information. Therefore a measure of uncertainty can be adopted as a measure of the system's effectiveness (performance): The lower is the uncertainty after performing an activity within the system, the better is the system's effectiveness with respect to this activity.

According to the standard probability theory the uncertainty of a non fuzzy system (and the information connected to it) is measured by the Shannon's formula [8], better known as *the Shannon's entropy*. For use in a fuzzy environment, this measure is expressed in the form: H= $-\frac{1}{\ln n}\sum_{s=1}^{n} m_s \ln m_s$ ([3], p. 20), where m: U $\rightarrow$ [0, 1] is the membership function of the corresponding fuzzy set, $m_s = m(s)$ denotes the membership degree of the element s of the universal (crisp) set U in the corresponding fuzzy subset of U and n denotes the total number of the elements of U. In the above formula the sum is divided by the natural logarithm of n in order to be normalized. Thus H takes values within the real interval [0, 1].

We recall that the *fuzzy probability* of an element s of U is defined in a way analogous to the crisp probability, i.e. by $P_s = \frac{m_s}{\sum_{s \in U} m_s}$. However, according to Shackle [7] and many other researchers after him, human reasoning can be formulated more adequately by the possibility rather, than by the probability theory. The *possibility* r_s of s is defined by $r_s = \frac{m_s}{\max\{m_s\}}$, where $\max\{m_s\}$ denotes the maximal value of m_s, for all s in U. In other words, the possibility of s expresses the relative membership degree of s with respect to $\max\{m_s\}$. Following Shackle's view we shall use the possibilities instead of probabilities in measuring a system's uncertainty..

Within the domain of possibility theory uncertainty consists of *strife (or discord)*, which expresses conflicts among the various sets of alternatives, and *non-specificity (or imprecision)*, which indicates that some alternatives are left unspecified, i.e. it expresses conflicts among the cardinalities of the various sets of alternatives ([3]; p.28). For a better intuitive understanding of the above two types of uncertainty we present the following simple example:

EXAMPLE: Let U be the set of integers from 0 to 100 and let Y = young, A = adult and O = old be fuzzy subsets of U defined by the membership functions m_Y, m_A and m_O respectively, where people are considered as young, adult or old according to their outer appearance . Then, given x in U, there usually exists a degree of uncertainty about the reasonable values that the membership degrees $m_Y(x)$, $m_A(x)$ and $m_O(x)$ could take, resulting to a conflict among the fuzzy subsets Y, A and O of U. For instance, if x = 18, values like $m_Y(x)$ = 0.8 and $m_A(x)$ = 0.3 are acceptable, but they are not the only ones! In fact, the values $m_Y(x)$ = 1 and $m_A(x)$ = 0.5 are also acceptable, etc. The existing conflict becomes even higher, if x =50. In fact, is it reasonable in this case to take $m_Y(x)$ =0? Probably not, because sometimes people being 50 years old look much younger than others aged 40 or even 30 years. But, there exist also people aged 50 who look older from others aged 70, or even 80 years! So what about the acceptable values of $m_O(x)$? All the above are examples of the type of uncertainty that we have termed as strife. On the other hand, non - specificity is connected to the question: How many x in U should have non zero membership degrees in Y, A or O respectively? In other words, the existing in this case uncertainty creates a conflict among the cardinalities (sizes) of the fuzzy subsets of U. We recall that the *cardinality* of a fuzzy subset, say B, of U is defined to be the sum $\sum_{x \in U} m_B(x)$ of all membership degrees of the elements of U in B. -

Strife is measured by the function $ST(r)$ on the ordered possibility distribution r: $r_1=1 \geq r_2 \geq \ldots\ldots \geq r_n \geq r_{n+1}$ of the elements of U with respect to the corresponding fuzzy subset of U defined by

$$ST(r) = \frac{1}{\log 2}\left[\sum_{i=2}^{m}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}\right] \quad ([3], \text{p.28}).$$

Similarly non-specificity is measured by the function

$$N(r) = \frac{1}{\log 2}\left[\sum_{i=2}^{m}(r_i - r_{i+1})\log i\right] \quad ([3], \text{p.28}).$$

The sum $T(r) = ST(r) + N(r)$ measures the *total possibilistic uncertainty* for ordered possibility distributions. The lower is the value of $T(r)$, which means greater reduction of the initially (before the activity) existing uncertainty, the better is the system's performance with respect to this activity (see also [], section 2).

3. The centroid defuzzification technique

An alternative and very popular method for assessing a fuzzy system's performance is the use of the *centroid defuzzification technique,* briefly denoted as COG, an acronym created by the initials of the words "**C**entre **O**f **G**ravity'. In applying the COG method we correspond to each $x \in$ U an interval of values from a prefixed numerical distribution, which actually means that we replace U with a set of real intervals. Then, we construct the graph of the membership function $y=m(x)$. There is a commonly used in fuzzy logic approach (e.g. see [10]) to measure performance with the pair of numbers (x_c, y_c) as the coordinates of the centre of gravity (centroid), say F_c, of the level's section F contained between the above graph and the OX axis, which we can calculate using the following well-known from Mechanics formulas:

$$x_c = \frac{\iint_F x\,dxdy}{\iint_F dxdy},\ y_c = \frac{\iint_F y\,dxdy}{\iint_F dxdy} \qquad (1)$$

Let us now see how we can formulate the above technique for our purposes in this paper (see also [9], [14], [16],etc). For this, let us consider a group G of n students during an activity and let us characterize the students' performance with respect to this activity by the *fuzzy linguistic labels* low (E) if x $\in$ [0, 1), almost satisfactory (D) if x $\in$ [1, 2), good (C) if x$\in$ [2, 3), very good (B) if x $\in$ [3, 4) and excellent (A) if x $\in$ [4, 5] respectively. This could be achieved in practice by assigning a mark to each student within the interval [0, 5]. We set U = {A, B, C, D, E}. Obviously in this case the level's section F is the union of the five rectangles F_i, i=1, 2, 3, 4, 5, presented in Figure 3. The one side of each one of the above rectangles has length 1 and lies on OX.

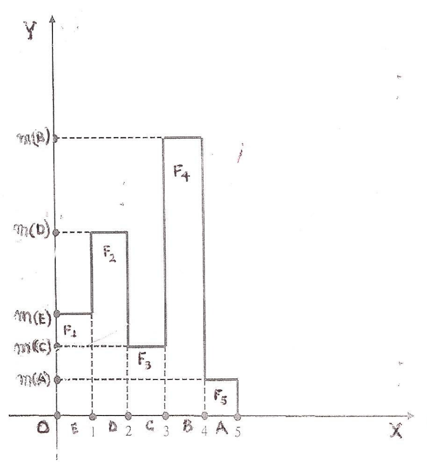

Figure 3: Graphical representation of the fuzzy data

It is straightforward then to check (e.g. see section 3 of []) that in this case formulas (1) can be transformed to the form:

$$x_c = \frac{1}{2}\left(y_1 + 3y_2 + 5y_3 + 7y_4 + 9y_5\right),$$
$$y_c = \frac{1}{2}\left(y_1{}^2 + y_2{}^2 + y_3{}^2 + y_4{}^2 + y_5{}^2\right) \quad (2)$$

with $y_i = \frac{m(x_i)}{\sum_{x \in U} m(x)}$, where $y_1 = m(E)$, $y_2 = m(D)$,, $y_3 = m(C)$, $y_4 = m(B)$ and $y_5 = m(A)$.

Then, using elementary algebraic inequalities it is easy to show that there is a unique minimum for y_c corresponding to the centre of gravity $F_m\left(\frac{5}{2}, \frac{1}{10}\right)$. Further, the ideal case for the student group's performance is when $y_1=y_2=y_3=y_4=0$ and $y_5=1$. Replacing the above values of y_i's to formulas (2) we find that this case corresponds to the center of gravity $F_i\left(\frac{9}{2}, \frac{1}{2}\right)$. On the other hand the worst case is when $y_1=1$ and $y_2=y_3=y_4=y_5=0$. This case, as it turns out from formulas (2) again, corresponds to the centre of gravity $F_w\left(\frac{1}{2}, \frac{1}{2}\right)$.

Therefore the centre of gravity F_c lies in the area of the triangle $F_w F_m F_i$. Then by elementary geometric considerations one obtains (e.g see section 3 of [16]) the following criterion for comparing the groups' performances:

- *Among two or more groups the group with the greater value of x_c performs better.*
- *If two or more groups have the same $x_c \geq 2.5$, then the group with the greater value of y_c performs better.*
- *If two or more groups have the same $x_c < 2.5$, then the group with the smaller value of y_c performs better.*

As it becomes evident from the above description, the application of the COG method is simple in practice and, in contrast to the measurement of the system's total possibilistic uncertainty, needs no complicated calculations in its final step.

However, we must emphasize that the COG method treats differently the idea of a system's performance, than the measurement of the uncertainty does. In fact, as it can be easily observed by formulas (2), the weighted average plays the main role in the COG method, i.e. the result of the system's performance close to its ideal performance has much more "weight" than the one close to the lower end. In other words, while the measurement of uncertainty is connected to the *average system's performance*, the COG method is mostly looking at the *quality of the performance*. Consequently, in using these two methods to compare the performances of two different fuzzy systems (e.g. student groups') with respect to the same activity, it is possible for the system demonstrating the better mean performance to demonstrate a worse quality performance than the other one. Therefore, it is argued that a combined use of these two methods could help the user in finding the ideal profile of the system's performance according to his/her personal criteria of goals.

4. CT: A classroom experiment

Living in a knowledge era and an ever increasing progress in technology, combining knowledge and technology to solve problems is becoming the mode rather than the exception. But, as we have already seen in our introduction, if technology is added as another tool, then CT is a prerequisite for PS. On the other hand, mathematics by its nature is a subject whereby PS forms its essence. Therefore the use of computers as a tool for teaching mathematics is connected to the students' CT skills for PS. The following experiment, an innovation of which is that we have applied as assessment methods the two fuzzy sets methods presented in the previous sections, comes to support/extend the already reported in the literature experimental results (based on principles of the traditional logic), concerning the beneficial influence of the use of computers in enhancing students' PS skills (see our introduction)..

Description of the experiment

The experiment was recently performed in the city of Patras within the course of “Higher Mathematics I”, which is attached to the first term of studies of the School of Technological Applications (prospective engineers)[1] of the Graduate Technological Educational Institute (T. E. I.) of Western Greece. The topics of this course include Complex Numbers, Linear Algebra, Elements of Analytic Geometry Differential and Integral Calculus in one variable and Elementary Differential Equations. The subjects were 90 students of the Department of Civil Engineering, who were divided in two equivalent groups (according to their marks obtained in the Panhellenic mathematics exam for entrance to the Higher Education) of 45 students in each group. The students of both groups had no previous experience with computers, apart from the basics learned in High School.

For the control group the teaching process was performed in the traditional way (theory, exercises and PS on the board) including a series of examples on mathematical modelling connecting the teaching material with real world problems. The students participated actively in the solution of exercises and problems. The difference with the experimental group was that about the one third of the teaching process, whose total duration was the same with that of the control group, took place in a computers’ laboratory. There, the tutor was trying to give to students with the help of technology a first, mainly intuitive, understanding of the corresponding theory. Next, the students divided in small groups used, under the tutor’s supervision, the proper mathematical software in solving relevant exercises and problems. The teaching procedure was completed later in the classroom with the typical proofs given on the board. Next, the students, after discussing with the tutor their right and wrong reactions during the laboratory activities, they solved similar problems and exercises by hand.

[1] The Departments of Engineering of the T. E. I. ‘s are similar to the corresponding Departments of the Polytechnic Schools of the Universities. Notice that in Greece both Universities and T. E. I. ‘s belong to the Higher Education., but while the T. E. I.’ s are mainly oriented to the field of applications and technological research, a more theoretical emphasis is given at the Universities towards the basic and/or applied research. For more details see [11].

Finally, and through a new discussion conducted by the tutor, the final conclusions were drawn.

At the end of the term the students of both groups participated in the course's final written exam. The first part of this exam included a number of theoretical questions and exercises of application covering all the topics taught, while the second part included three real world problems involving mathematical modelling in their solutions. The two parts of the exam were marked separately and the mean of the two marks was each student's final mark for the exam (for the final students' assessment their progress marks obtained during the course were also considered in a 30% percentage).

Assessing the two groups' total performance

In representing the two student groups as fuzzy subsets of U = {A, B, C, D, E} we defined the membership function m: U $\rightarrow$ [0, 1] as follows:

$$y = m(x) = \begin{cases} 1, & if \quad 80\% \ n < n_x \le n \\ 0{,}75, & if \quad 50\% \ n < n_x \le 80\% \ n \\ 0{,}5, & if \quad 20\% \ n < n_x \le 50\% \ n \\ 0{,}25, & if \quad 1\% \ n < n_x \le 20\% \ n \\ 0, & if \quad 0 \le n_x \le 1\% \ n \end{cases} ,$$

for each x in U, where n = 45 is the number of student's in each group and n_x denotes the number of each group's students whose performance was characterized by the linguistic label x. The above definition was based on our past assessment experiences and it is obviously compatible to the common logic. Then each group G can be written as a fuzzy subset of U in the form G = {(x, m(x)): x$\in$ U}.

The fuzzy data of our classroom experiment is presented in Tables 1 and 2 below, where the students' scores obtained by marking their papers in the usual way.

Table 1: Theoretical questions and exercises

Experimental group (G_1)

% Scale	Performance	Number of students	*m(x)*
89-100	A	0	0
77-88	B	6	0.25
65-76	C	8	0.25
50-64	D	19	0.5
Less than 50	E	12	0.5
Total		45	1.5

Control group (G_2)

% Scale	Performance	Number of students	*m(x)*
89-10	A	0	0
77-88	B	5	0.25
65-76	C	7	0.25
50-64	D	20	0.5
Less than 50	E	13	0.5
Total		45	1.5

It can be easily observed that the membership degrees in Table 1 for each x in U are the same for each group, therefore the order possibility distribution of the two groups is also the same. Thus, the two groups demonstrated the same mean (measure of uncertainty) and weighted performance (COG method) at the first part of the exam (theoretical questions and exercises).

From Table 2 (see below) we find that $max\{m_x\}$ = 0.5 for both groups, therefore the possibilities of the elements of U are calculated by the formula $r_x = \frac{m_x}{0.5}$ for both groups. Performing the corresponding calculations, we find that $r_1=r_2=r_3=1$, $r_4=r_5=0.5$ for G_1 and $r_1=r_2=r_3=1$, $r_4=0.5$, $r_5=0$ for G_2. From the above values of possibilities and according to the corresponding formulas of section 2 we find for G_1 that

S(r)=$\frac{1}{\log 2}[\sum_{i=2}^{4}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}] = \frac{1}{\log 2} * 0.5 * \log\frac{3}{3}$ =0 and

N(r)=$\frac{1}{\log 2} * 0.5 * \log 3$ $\approx$0.792=T(r). Similarly we find for G_2 that S(r) =

$\frac{1}{\log 2} * 0.25 * \log\frac{4}{3.5} \approx 0.048$ and

N(r) = $\frac{1}{\log 2} * (0.5 * \log 3 + 0.25 * \log 4) \approx 1.292$, therefore T($r$) ≈ 0.048 +1.292 = 1.34.

Table 2: Problems

Experimental group (G_1)

% Scale	Performance	Number of students	*m(x)*
89-100	A	2	0.25
77-88	B	10	0.5
65-76	C	11	0.5
50-64	D	15	0.5
Less than 50	E	7	0.25
Total		45	2

Control group (G_2)

% Scale	Performance	Number of students	*m(x)*
89-100	A	0	0
77-88	B	8	0.25
65-76	C	12	0.5
50-64	D	15	0.5
Less than 50	E	10	0.5
Total		45	1.75

In applying the results obtained in section 3 we must first normalize the membership degrees. Thus, for G_1 we find $x_c = \frac{1}{2*2}(0.25+3*0.5+5*0.5+7*0.5+9*0.25) = \frac{10}{4} = 2.5$, while for G_2 we find

$$x_c = \frac{1}{2*1.75}(0.5+3*0.5+5*0.5+7*0.25) = \frac{6.25}{3.5} \approx 1.786 .$$

Therefore, while the two groups demonstrated the same mean and the same weighted performance for the first part of the exam, both the mean and the weighted performance of the experimental group were found to be significantly better than the corresponding performances of the control group in the second part (problems). This outcome gives a strong indication that the use of computers in teaching mathematics enhances the students' abilities in modelling and solving mathematically real world problems.

5. Conclusions and discussion

CT is a term recently introduced to describe a set of thinking skills that are integral for systems' analysis, for solving complex problems, and for the understanding of the human behavior, by drawing on principles fundamental to computer science. .In this article we have applied two frequently used in the fuzzy sets theory assessment techniques for studying the effects of CT in learning mathematics: The measurement of a fuzzy system's total possibilistic uncertainty, connected to the mean students' performance and the COG method (properly adapted for our purposes), connected to the students' quality performance.

The results of our classroom experiment, performed with students of the Graduate T. E. I. of Western Greece, provided a strong indication that CT enhances the students' skills in modelling and solving mathematically real world problems. This outcome supports/extends already reported in the literature experimental results (obtained with traditional methods) concerning the beneficial influence of the use of computers on students' PS skills. In fact, it seems that the figures' animation, the quick transformations of the numerical and algebraic representations, the easy and accurate construction of the several graphs especially in the 3-dimensional space, etc, which are comfortably achieved by using the proper mathematical software, increase the students' imagination and help them in using their intuition more effectively for designing/constructing the solutions of the corresponding problems. The role of the mathematical theory after this process is not anymore to convince, but simply to explain.

However, in obtaining statistically safer conclusions, further experimental research is required on the subject, e.g. by performing similar experiments with post-graduate and research students, with pupils of secondary and possibly of primary (upper classes) education, with prospective and in-service teachers of mathematics, etc. On the other hand, due to their generality, the assessment methods applied in this paper could by used in future for several other human activities, including student courses of other disciplines where PS plays an important role (e.g. physics, engineering, economics, etc), but also sports and spiritual games, decision making, artificial intelligence, etc

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