An Approach to Goldbach's Conjecture

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Abstract

This paper attempts to prove Goldbach's conjecture. As its major contribution, an alternative perspective to the proof of Goldbach' conjecture is presented.

Keywords: division algorithm, circle method, primes.

1. Introduction

In 1741, a profound contribution in mathematics was made by Christian Goldbach with the conjecture that all even numbers can be expressed as the sum of two primes [2]. As at the time Goldbach lived, the number 1 was considered a prime number. Currently, the conjecture could be stated as "every even positive integer greater than 3 is the sum of (not necessarily distinct) primes".

Goldbach's conjecture is one of the most famous and difficult problems in mathematics. Since it was posed, mathematicians have made great attempt to provide its proof and thereby, developed several number-theoretic methods. One of such methods is the *circle method*, introduced by Hardy and Littlewood in 1923 that *every sufficiently large odd integer is the sum of three prime and almost all even integers is the sum of two primes provided the grand Riemann hypothesis is assumed to be true* [3]. Another result propelled by Goldbach's conjecture is the one published in 1919 by the Norwegian mathematician Brun that *every large even number is the sum of two numbers each having at most* *nine prime factors*. Recently, Miles Mathis (see milesmathis.com/gold3) claimed to have solved Goldbach's conjecture by calculating probabilities for primes and non-prime meetings. In his approach, probability fractions were redefined and transformed into densities which allow a proof free of probabilities. However, researchers have pointed out an apparent contradiction in his approach; that switching mid-problem from fraction of terms to fraction of odds goes against his own rule [4].

In the work of Bernard Farley (2005), two approaches, namely, (i) the sequence approach and (ii) the counting approach to Goldbach's conjecture are demonstrated [1]. Essentially, an *equivalence* statement is used to tackle Goldbach's conjecture (see www.math.vt.edu).

In this paper, we present a straight forward proof to Goldbach's conjecture. Although the nature of our proof is the known *prove by contradiction* technique, yet, it differs from other published proves, both in construct and simplicity.

2. Basic concepts

In number theory, it is known that for every three consecutive even numbers, one of them is a multiple of 3, so we could write $3.2n = 6n$ or even obtain a triple of the form: $(6n, 6n - 2, 6n + 2)$. In section 3, we will use this idea to construct our proof. Also, it is important to state here that in the construction of our proof, we will use the well-known theorem: *every even number 2n can be decomposed into the sum of two odd numbers ------------- (*).*

Further, as a way of recalling, we state the following:

The sum of two even numbers is even ------------------------------- (**).

This means that every even number can be written as the sum of two even numbers.

As of now, it is important to note that even numbers cannot be written as the sum of an even and an odd number.

3. The proof

Goldbach's conjecture stated in the first paragraph of section 1 could be verified manually as shown below, but the difficulty in this verification is that it continues indefinitely. This is why we need a formal proof.

Verification: $4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7, 14 = 7 + 7,$ and so on.

As simple as it may appear, Goldbach's conjecture: $2n = p + q, \forall n \in \mathbb{N}, and\ where\ p\ and\ \ q\ are\ primes\ (depending\ on\ n)$ has shown itself to be quite difficult to prove. In what follows, we present a proof to this conjecture.

Proof.

Let k be an even positive integer greater than 3. Suppose k cannot be expressed as the sum of two prime numbers. By *well ordering of* $\mathbb{N}$, there exist $p, q \in \mathbb{N}$ such that $p \leq q < k$ with $k = p + q$……………………………….…(1)

An implication of our supposition is thus;

I. Either p is a prime number and q is not a prime number or

II. $p\ and\ q$ are not prime numbers.

<u>Case I.</u>

If p is a prime number and q is not a prime number, since k is even, we write:

$2n = p + q, \forall n \in \mathrm{N}$ …………………………….. (2)

From (**), we suppose that $p \; and \; q$ are both even numbers, then p must be equal to 2, since 2 is the only even prime number. In this instance, we have; $2n = 2 + q$ which reduces to $q = 2(n-1)$. But for $n = 2$, $q = 2 -$ a prime number. This contradicts our initial supposition that q is not a prime number.

Now, from (*), we suppose that $p \; and \; q$ are both odd numbers. This means p is of the form: $2m \pm 1, m = 0, 1, 2, 3, \ldots$. Using $p = 2m - 1$ in (2), we have the following:

$2n = 2m - 1 + q$ which simplifies to $q = 2r + 1$, where $r = n - m$, with $r = 0, 1, 2, \ldots$

It is easy to see that when $n > m$, q is prime for $r = 1, 2, 3, 5, 6, 8, \ldots$

Again, this is a contradiction to the supposition that q is not a prime number.

<u>Case II</u>

Suppose $p \; and \; q$ are not prime numbers which satisfy (2), we assume that assume that $p \; and \; q$ are both even numbers. This means that $p \geq 4$ and $q \geq 4$. Then we can find non-negative integers u and u' such that $p = 3u + v$ and $q = 3u' + v'$, where v and v' are remainders which can take one of the values 0, 1, or 2.

From (2), we have; $2n = 3u + v + 3u' + v'$ ………………………………(3)

where $(u = u' = 1, 2, 3, \ldots; v = v'$ equal one of the values $0, 1, or \; 2$. By substituting the initial values of u, u', v and v' in (3), it is clear that $2n$ is the sum of (not necessarily distinct) primes. This contradicts our initial supposition.

Finally, we assume that p and q are both odd non-prime numbers satisfying (2).Then p and q are values from $9, 15, 21, \ldots$ and above. From division algorithm, there exist non-negative integers a and a' such that $p = 6a + b$ and $q = 6a' + b'$, where b and b' are remainders which can take one of the values 1 or 3. And so, $2n = p + q$ can be written as:

$2n = 6a + b + 6a' + b'$ …………………………… (4)

where $(a = a' = 2, 3, 4 \ldots; b = b'$ equal one of $1 \, or \, 3$. By substituting the initial values of a, a', b and b' in (4), it is evident that $2n$ is the sum of (not necessarily distinct) primes. Since p and q depend on n, the result follows immediately.

4. Concluding remarks

This paper has presented a simplified approach to Goldbach's conjecture. Thus, providing a non-complicated proof to the *simple at a glance* statement made by Christian Goldbach (1690- 1764). Some interesting things about this approach are that it employs simple concept of number theory, it is not internally contradictory and admits notions that could be easily verified by both mathematicians and non-mathematicians.

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Reference

[1] Bernard, F. Two Approaches to Proving Goldbach's Conjecture, www.math.vt.edu. (2005).

[2] Goldbach, C. Letter to L. Euler, June 7, 1742.

[3] Hardy, G. H. and Littlewood, J. E. ”Some Problems of Partitio Numerorum (V): A Further Contribution to the Study of Goldbach’s Problem.” Proc. London Math. Soc. Ser. 2 22, 46-56, 1924.

[4] Miles, M. The Simple Proof of Golddbach’s Conjecture, www.milesmathis.com/gold3, (2014).