Application of Fuzzy Numbers for Assessing Student Learning with the Bloom's Taxonomy

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Abstract

In the paper at hand Triangular Fuzzy Numbers are utilized for developing an assessment method of a student group performance, which is applied for evaluating student learning of the real numbers in terms of the Bloom's Taxonomy. The outcomes of this fuzzy assessment method are compared to the traditional calculation of the mean value of student scores.

Keywords: Bloom's Taxonomy, Fuzzy Assessment Methods, Triangular Fuzzy Numbers (TFNs), The Process of Learning, Teaching the Real Numbers.

1. Introduction

The knowledge that students have about various concepts is usually imperfect characterized by a different degree of depth. On the other hand, from the teacher's point of view there exists frequently an uncertainty about the degree of acquisition of a subject matter by students. All these gave the hint to the present author to introduce in earlier works principles of Fuzzy Logic (FL) for a more effective description of the process of learning [21, 23: Chapter 2, 26: Section 4.5] and for the assessment of student learning skills [19, 22, 24], etc.

In the paper at hand triangular fuzzy numbers are used as a tool for developing a new assessment method of student learning in lines of the Bloom's taxonomy. The rest of the paper is formulated as follows: In Section 2, after a general introduction to the process of learning, the main principles of the Bloom's taxonomy are presented. In Section 3 a method of using triangular fuzzy numbers as tools for assessing a student group performance is described, while in Section 4 an example is presented on student learning of the real numbers illustrating our results. The paper closes with Section 5, which is devoted to our conclusion and to some hints for future research. .

2. The Bloom's Taxonomy for Teaching and Learning

Learning can be commonly defined as the activity of gaining knowledge or skill. The ability to learn is possessed by humans, animals, plants [12] and computers [16]. Learning does not happen all at once, but it builds upon and is shaped by previous knowledge. To that end, learning may be viewed as a process, rather than a collection of factual and procedural knowledge. In psychology and education learning refers to a process that brings together cognitive, emotional and environmental influences and experiences for acquiring, enhancing or making changes in one's knowledge, skills, values and world views [15].

The process of learning is fundamental to the study of human cognitive action. Volumes of research have been written about it and many attempts have been made by psychologists, cognitive scientists and educators to make learning accessible to all in various ways. There are three main philosophical frameworks under which learning theories fall: ***Behaviorism***, ***Cognitivism*** and ***Constructivism***. Behaviorism focuses only on the objectively observable aspects of learning; for behaviorists learning is the acquisition of new behavior through conditioning. Cognitive theories look beyond behavior to explain brain-based learning, while constructivism views learning as a process in which the learner actively constructs or builds new ideas and concepts.

Over the last four decades mathematics education has addressed philosophical and epistemological perspectives with respect to mathematics learning. It has become common to think of learning mathematics in fallibilistic terms [7, 8, 17], to consider learning as a problem-solving [27] or a constructive process [6, 20], to situate knowledge and learning relative to communities of practice [13] and to debate the commensurability of constructivist and socio-cultural learning theories [14, 18]. Theoretical considerations like the nature of mathematical knowledge, what it means to know mathematics and to come to know it, how knowing in mathematics is related to knowing in social settings more widely, have been deeply considered and seriously debated [2, 4, 5, 10]. The mathematics education discipline has become mature in such theoretical considerations.

In 1956 Benjamin Bloom with collaborators Max Englehart, Edward Furst, Walter Hill, and David Krathwohl published a framework for categorizing educational goals, the ***Taxonomy of Educational Objectives*** [3] [1]. Although named after Bloom, the publication of the taxonomy followed a series of conferences from 1949 to 1953, which were designed to improve communication between educators on the design of curricula and examinations. A revised version of the taxonomy was created in 2000 by Lorin Anderson [1], former student of Bloom. Since the taxonomy reflects different forms of thinking and thinking is an active process, in the revised version the names of its six major levels were changed from noun to verb forms. The six major levels of the revised taxonomy are presented in Figure 1, taken from [28].

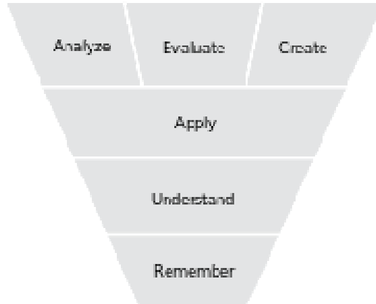

Figure 1: *The six major levels of the Bloom's taxonomy*

The above six levels in the taxonomy, moving through the lowest order processes to the highest, could be described as follows :

- ***Knowing - Remembering:*** Retrieving, recognizing, and recalling relevant knowledge from long-term memory, e.g. find out, learn terms, facts, methods, procedures, concepts

- ***Organizing - Understanding:*** Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Understand uses and implications of terms, facts, methods, procedures, concepts.

- ***Applying:*** Carrying out or using a procedure through executing, or implementing. Make use of, apply practice theory, solve problems, use information in new situations.

- ***Analyzing:*** Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing. Take concepts apart, break them down, analyze structure, recognize assumptions and poor logic, evaluate relevancy.

- ***Generating - Evaluating:*** Making judgments based on criteria and standards through checking and critiquing. Set standards, judge using standards, evidence, rubrics, accept or reject on basis of criteria.

- ***Integrating - Creating:*** Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing. Put things together; bring together various parts; write theme, present speech, plan experiment, put information together in a new & creative way

Most researchers and educators consider the last three levels --analyzing, evaluating and creating – as being parallel, i.e. as happening together. It is obvious that using Bloom's higher levels helps the students become better problem solvers.

For teaching a topic, the instructor should arrange his/her class work in the order to synchronize it with these six steps of Bloom's Taxonomy. The typical questions for evaluating the student achievement at the corresponding level are the following:

Knowing questions focus on clarifying, recalling, naming, and listing:
Which illustrates...?
Write... in standard form....
What is the correct way to write the number of... in word form?

Organizing questions focus on arranging information, comparing similarities/ differences, classifying, and sequencing:
Which shows... in order from...?
What is the order...?
Which is the difference between a... and a...?
Which is the same as...?
Express... as a...?

Applying questions focus on prior knowledge to solve a problem:
What was the total...?
What is the value of...?
How many... would be needed for...?
Solve....Add/subtract....Find....Evaluate....Estimate....Graph....

Analyzing questions focus on examining parts, identifying attributes/ relationships /patterns, and main idea:
Which tells...?
If the pattern continues, ...
Which could...?
What rule explains/completes... this pattern?
What is/are missing?
What is the best estimate for...?
Which shows...?
What is the effect of...?

Generating questions focus on producing new information, inferring, predicting, and elaborating with details:
What number does... stand for?
What is the probability...?
What are the chances...?
What effect...?

Integrating questions focus on connecting/combining/summarizing information, and restructuring existing information to incorporate new information:
How many different...?
What happens to... when...?
What is the significance of...?
How many different combinations...?
Find the number of..., ..., and ... in the figure below.

Evaluating questions focus on reasonableness and quality of ideas, criteria for making judgments and confirming accuracy of claims:
Which most accurately...?
Which is correct?
Which statement about... is true?
What are the chances...?
Which would best...?
Which would... the same...?
Which statement is sufficient to proven...?

Bloom's taxonomy serves as the backbone of many teaching philosophies, in particular those that lean more towards skills rather than content. The emphasis on higher-order thinking inherent in such philosophies is based on the top levels of the taxonomy including analysis, evaluation, synthesis and creation. Bloom's taxonomy can be used as a teaching tool to help balance assessment and evaluative questions in class, assignments and texts to ensure all orders of thinking are exercised in student's learning.

3. Assessment of Student Group Performance Using Triangular Fuzzy Numbers

It is recalled that a ***Fuzzy Set (FS)*** A on the universal set of the discourse U (or a fuzzy subset of U) is a set of ordered pairs of the form $A = \{(x, m_A(x)): x \in U\}$, defined in terms of a ***membership function*** $m: U \to [0, 1]$ that assigns to each element of U a real value from the interval [0,1]. The value $m(x)$ is called the ***membership degree*** of x in A. The greater is $m(x)$, the better x satisfies the characteristic property of A. The definition of the membership function is not unique depending on the user's subjective data, which are usually based on statistical or empirical observations. However, a necessary condition for a FS to give a reliable description of the corresponding real situation is that the definition of its membership function is compatible to the common logic. For general facts on FSs we refer to the book [11].

A ***Fuzzy Number (FN)*** is a FS on the set $\boldsymbol{R}$ of the real numbers which is normal - i.e. there exists x in $\boldsymbol{R}$ such that $m(x) = 1$ - and convex - i.e. all it's a-cuts $A^a = \{x \in \boldsymbol{R}: m(x) \geq a\}$, with a in [0, 1] are closed real intervals - while its membership function $y = m(x)$ is piecewise continuous. For general facts on FNs we refer to the book [9].

In particular a ***Triangular Fuzzy Number (TFN)*** of the form (a, b, c), with a, b, c real numbers such that $a<c<b$, is a FN with membership function defined by

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \text{ or } x > c \end{cases}$$

Let A (a, b, c) and B (a_1, b_1, c_1) be two TFNs and let k be a positive real number. Then the ***sum*** A + B = $(a+a_1, b+b_1, c+c_1)$ and the ***scalar product*** $kA = (ka, kb, kc)$. Further, given the TFNs A_i, i = 1, 2,…, n , where n a non negative integer, $n \geq 2$, we define their ***mean value*** to be the TFN $A = \frac{1}{n}(A_1 + A_2 + \ldots + A_n)$ (1).

The first step for the assessment of a student group performance using TFNs involves the numerical evaluation of each student's individual performance in a climax from 0 to 100. In order to characterize this performance qualitatively we introduce the fuzzy linguistic labels (grades) A (85-100) = excellent, B (75-84) = very good, C (60-74) = good, D(50-59) = fair and F(0-49) = non satisfactory [2]. Next, we assign to each of the above grades a TFN denoted by the same letter as follows: A=(85, 92.5, 100), B=(75, 79.5, 84), C=(60, 67, 74), D=(50, 54.5, 59) and F=(0, 24.5, 49). Observe that the middle entry of each of those TFNs is equal to the mean value of the student scores attached to the corresponding grade. In this way a TFN can be assigned to each student assessing his/her individual performance. Therefore, it is logical to use the mean value M of all those TFNs for evaluating the student group overall performance.

In earlier works we have used the ***Centre of Gravity (COG)*** technique for the defuzzification of a TFN T = (a, b, c). This technique leads to the representation of T by the x-coordinate $x(T) = \frac{a+b+c}{3}$ (2) of the COG of its graph which is a triangle with vertices the points with coordinates $(a, 0)$, $(b, 1)$ and $(c, 0)$ respectively ([25], Proposition 1 of Section 3].

In particular, if T is one of the TFNs A, B, C, D, F then $b = \frac{a+c}{2}$. Therefore, equation (2) gives that x(T)= $\frac{a+\frac{a+c}{2}+c}{3}=\frac{3(a+c)}{6}=b$. But, by equation (1) the mean value M=k_1A+k_2B+k_3C+k_4D+k_5F, with k_i non negative rational numbers, i=1, 2, 3, 4, 5. Consequently, if A (a_1, b_1, c_1), B (a_2, b_2, c_2),…., F(a_5, b_5, c_5) and M(a, b, c), then M = $\sum_{i=1}^{5} k_i(a_i,b_i,c_i)=(\sum_{i=1}^{5} k_i a_i,\sum_{i=1}^{5} k_i b_i,\sum_{i=1}^{5} k_i c_i)$. Therefore, x(M) =

$$\frac{\sum_{i=1}^{5} k_i a_i+\sum_{i=1}^{5} k_i b_i+\sum_{i=1}^{5} k_i c_i}{3}=\sum_{i=1}^{5} k_i\frac{a_i+b_i+c_i}{3}=\sum_{i=1}^{5} k_i b_i=b$$

4. An application to teaching the real numbers

4.1 Description

The following application was performed with subjects the students of two different departments (30 students in each department) of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece attending the common course "Mathematics I" of their first term of studies and having the same instructor. This course involves an introductory module repeating and extending the students' knowledge from secondary education about the real numbers. After the module was taught, the instructor wanted to investigate the students' progress according to the principles of the Bloom's Taxonomy. For this, he asked them to answer in the class the written test presented in the Appendix of this paper, which is divided in six different parts, one for each level of the Taxonomy. The students' answers were assessed separately for each level in a scale from 0 to 100 and the means obtained correspond to each student's overall performance.

4.2 Results

Denote by L_i, i=1, 2, 3, 4, 5, 6 the levels of Knowing-Remembering, Organizing-Understanding, Applying, Analyzing, Generating-Evaluating and Integrating- Creating respectively of the Bloom's Taxonomy and by P the student overall performance. Then the test's results are depicted in the following two tables:

Table 1: Results of the first department

Grade	L_1	L_2	L_3	L_4	L_5	L_6	P
A(85-100)	8	6	5	3	2	3	4
B(84-75)	9	11	10	8	7	8	9
C(74-60)	10	9	10	12	10	8	10
D(59-50)	3	3	3	5	7	8	5
F(<50)	0	1	2	2	4	3	2

Table 2: Results of the second department

Grade	L_1	L_2	L_3	L_4	L_5	L_6	P
A(85-100)	9	8	6	4	3	3	5
B(84-75)	6	7	9	7	7	6	8
C(74-60)	9	8	10	12	10	8	9
D(59-50)	6	7	4	4	7	11	7
F(<50)	0	0	1	3	3	2	1

4.3 Evaluation of the results using FNs

For reasons of simplicity, let us denote the mean values of each department's performance at level L_i, i =1, 2,…, 6 and its overall performance P by the same letters. Then, from Table 1 one finds that for the first department $L_1 = \frac{1}{30}(8A+9B+10C+3D) = \frac{1}{30}(2105, 2287.5, 2473) \approx (70.17, 76.25, 82.43)$, which gives that $x(L_1) \approx 76.25$.

Working in the same way one also finds that $x(L_2) \approx 74.02$, $x(L_3) \approx 71.33$, $x(L_4) \approx 67.97$, , $x(L_5) \approx 63.33$, $x(L_6) = 65.3$ and $x(P) \approx 69.23$. The above outcomes show that the first department demonstrated a very good (B) performance at level L_1 of the Bloom's Taxonomy, a good (C) performance at all the other levels and a good overall performance as well.

Similarly, from Table 2 and for the second department one finds that $x(L_1) = 74.65$, $x(L_2) = 73.8$, $x(L_3) \approx 72.77$, $x(L_4) = 67.4$, $x(L_5) = 65.3$, $x(L_6) = 61.3$ and $x(P) =70.25$. The above outcomes show that the second department demonstrated a good performance at all levels of the Taxonomy and a good overall performance as well.

On comparing the outcomes of the two departments one concludes that the first department demonstrated a better performance at levels L_1, L_2, L_4 and L_6 of the Bloom's Taxonomy, while the second department demonstrated a better performance at levels L_3 and L_5 and a better overall performance than the first department.

Observe also that the performance of each department is decreasing from level L_1 to level L_4, which was expected, since the success at the higher levels is based on the lower levels. However, for the first department this does not happen for the last three levels, a fact which is compatible to the view of most researchers and educators that the three higher levels of the Taxonomy are parallel to each other

4.4 Comparison with the mean values

As it becomes evident from the description of the method presented in Section 3, the use of TFNs evaluates the ***mean performance*** of a student group. The corresponding traditional method of the bi-valued Logic is the calculation of the ***mean value*** of the student scores.

The data of Tables 1 and 2 are not sufficient for calculating the mean values of the student scores of the two departments. However, it is easy to observe that the outcomes of the two assessment methods (TFNs and mean values) may differ to each other. For example, in the hypothetical case where the students of the last column of Table 1 obtained the highest scores of the corresponding grade (i.e. 4 students scored 100, 9 students scored 84, etc), while the students of the last column of Table 2 obtained the lowest scores of the corresponding grade (i.e. 5 students scored 85, 8 students scored 75, etc), calculating the mean values one finds an average score 64.51 for the first and 53.33 for the second department. Therefore, in this case the first department demonstrates a much better mean overall performance than the second one, in contrast to the outcomes obtained in terms of the TFNs. This is due to the different philosophies of the traditional bi-valued logic and the multi-valued fuzzy logic.

5. Conclusion

Fuzzy logic, due to its property of characterizing the ambiguous cases with multiple values, offers rich resources for assessment purposes. In the paper at hand we utilized TFNs for developing an assessment method of a student group mean performance and we applied it for evaluating the student learning of the real numbers in terms of the Bloom's Taxonomy. The outcomes of this method may differ from the traditional calculation of the mean values of student scores, which is based on the principles of the classical bi-valued logic.

The general character of the above fuzzy assessment method enables one to apply it in other sectors of human activities and this could be one of the main targets of future research on the subject.

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Footnotes

[1] Bloom's taxonomy divides educational objectives into three domains: ***cognitive***, ***affective*** and ***psychomotor***, sometimes loosely described as "knowing/head", "feeling/heart" and "doing/hands" respectively. The volume published in 1956 [5] and the revision followed in 2000 [6] concern the cognitive domain, while a second volume published in 1965 on the affective domain. A third volume was planned on the psychomotor domain, but it was never published. However, other authors published their own taxonomies on the last domain. More details can be found in [28].

[2] Obviously, the above assignment of the student scores to the corresponding qualitative is not unique. For example, in a more strict assessment one could take A(90-100), B (80-89), C (70-79), D (60-69) and F (<60). Also, one could add more qualitative grades, e.g. inserting E (marginal success) between D and F, etc.

Appendix: The questionnaire used in our application

Topic: Real numbers (introductory College level)

1. Knowing - Remembering

- Give the definitions and examples of a periodic decimal and of an irrational number (in the form of an infinite decimal).

2. Organizing

- Compare the set of all fractions with the set of periodic decimals. Compare the set of irrational numbers with the set of all roots (of any order) that have no exact values.

3. Applying

- Which of the following numbers are natural, integers, rational, irrational and real numbers?

-2 , $-\frac{5}{3}$, 0 , 9.08 , 5 , $7.333...$, $\pi = 3.14159...$, $\sqrt{3}$, $-\sqrt{4}$, $\frac{22}{11}$, $5\sqrt{3}$, $-\frac{\sqrt{5}}{\sqrt{20}}$, $(\sqrt{3}+2)(\sqrt{3}-2)$, $-\frac{\sqrt{5}}{2}$, $\sqrt{7}-2$, $\sqrt{\left(\frac{5}{3}\right)^2}$

4. Analyzing

- Find the digit which is in the 1005th place of the decimal 2.825342342......
- Write the number 0.345345345... in its fractional form.
- Compare the numbers 5 and 4.9999…
- Construct the line segment of length $\sqrt{3}$ with the help of the Pythagorean Theorem. Give a geometric interpretation.

5. Generating- Evaluating

- Justify why the decimals 2.00131311311131111... and 0.1234567891011... are irrational numbers.
- Construct the line segment of length $\sqrt[3]{2}$ by using the graph of the function f(x)= $\sqrt[3]{x}$

6. Integrating- Creating:

- Define the set of the real numbers in terms of their decimal representations (this definition was not given by the instructor to the class before the test).