Amalgamated Philosophical Structures: Dynamics in Cognitive Universal Algebra

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Abstract

We apply theory from Universal Algebra and Category Theory to investigate some cognitive implications for individuals or communities possessing amalgamated philosophies. Different groups of people having different philosophies, religions, theories, or ideologies may be challenged when confronted with working together to solve a common problem. Does the mere amalgamation (gluing) of philosophies, religions, theories, or ideologies of respective groups of people guarantee that they will work together? While a universal property of such an amalgamation (gluing) does logically guarantee the opportunity for mutual methodology during intergroup problem solving, various human factors can impede the implementation of mutual methodologies afforded by the universal property. This work also outlines some emotional and contextual impedances to mutual methodology implementation.

Keywords: mathematical sociology, mathematical psychology, philosophy-amalgamation, cognitive structures, universal property, universal algebra, category.

1. Introduction

Undoubtedly, philosophies, theories, religions, ideologies, or value systems are formed and reside in cognitive substructures of the human mind. These philosophies, theories, etc. are cognitive structures formed as a result of perception, learning, and the imagination [1]. Without loss of generality, we consider that all philosophies, theories, etc., have "basic" principles, axioms, beliefs, definitions, or theorems that are the cognitive elements under the psychology of the person bearing such philosophy. Also, in the sense of a cognitive set containing a "complete" set of principles, axioms, beliefs, definitions, or theorems; we will denote as a philosophy, any theory, religion, ideology, or value system. To the degree that social dynamics are importantly dependent on individual psychological dynamics, this work explores the implications of intergroup or inter-community joint problem solving, given the presence of amalgamated philosophical structures.

Amalgamation or "gluing" of spaces or structures is a common construction in mathematics. Topological spaces X and Y can be glued along respective subsets via some continuous map $f: X \to Y$ [2]. A CW-complex X^n can be constructed by properly attaching (gluing) the $(n-1)$- cells from X^{n-1} [3]. In the case where X and Y are algebras, they can be amalgamated along some ideal I in Y with respect to a homomorphism $h: X \to Y$ [4]. Under Category theory, cognitive constructions other than amalgamation (gluing), have been explored by Phillips and Wilson [5]. In a mental categorical-space, they applied category theory to explain the role of cognitive categorical Products in *transitive-inference* decisions and Co-products in *class-inclusion* decisions. Subjects that were able to make correct decisions involving *transitive-inference* were able to construct Products and subjects that were able to make correct decisions involving *class-inclusion* were able to construct Co-products. The study also found that the ability for subjects to make such cognitive constructions, Products or Co-products, is age sensitive. Most children below the age of five could not construct cognitive Products or Co-products. However, the universal property intrinsic to both of these structures, Products and Co-products, implies that they are isomorphic. Through this natural cognitive isomorphism, *cognitive systematicity* emerges; that is, subjects that were able to make correct decisions involving *transitive-inference* from Products, also have the ability to construct Co-products to make decisions involving *class-inclusion*. George Boole, in his book An Investigation of the Laws of Thought…, characterized the mind as a ring where the two ring operations commute over each other and are idempotent. With this ring structure, Boole sought to "investigated the fundamental laws of the operations of the mind by which reasoning is performed" [6].

In our work, the cognitive space is a substructure of the mind, and is given the structure of an abstract complex that is also an Algebra. Its cognitive entities are generalized abstract simplexes, representing given philosophies, with the assumption that the principles of the respective philosophies are formed into formulas (statements). The mind can cognitively form formulas of its principles, "combined" and "punctuated" under some cognitive signature ("grammar rules") of the mind. We apply concepts from Universal Algebra to develop criteria for "gluing" philosophies, via a sematic preserving homomorphism. Glued topological/algebraic structures possess a Universal Property. We investigate the cognitive implications for individuals or communities possessing "glued" philosophies. Cognitive explanations are enriched by geometric interpretations of the simplex.

In the following sections, we provide the nomenclature and theory for gluing structures, supported by psycho-social examples.

2. Geometric Simplex Representation

In general, a simplex is a structure A that is the "span" of a set of "objects", finite or infinite. The objects can be any defined entity, and span can mean any combination of the objects defined by the "grammar" of the structure. An abstract simplex can be realized by a Geometric simplex [7]. Here we provide the definition of a geometric simplex to facilitate our explanation on gluing philosophies.

Definition 1. (Geometric Simplex) A simplex, $\boldsymbol{S_p}$, is a system that consists of points spanned by vertices. Let $P = \{a_0, a_1, a_2, \dots, a_N\}$, be an independent set of vertices a_j. If b is a point or element in the simplex S_p spanned by P , then $b = \sum_{j=0}^{N} \lambda_j a_j$ where $0 \leq \lambda_j \leq 1$, and $\sum_{j=0}^{N} \lambda_j = 1$.

Definition 2. Any face E of a simplex $\boldsymbol{S_p}$ is a space spanned by a non-empty proper subset of its vertices $\boldsymbol{P}$, written, $E \prec \boldsymbol{S_p}$.

In [8] Kee et. al. applied the theory of a simplicial complex to model social aggregation, and in [9] Legrand applied Q-analysis was over a simplicial complex to study social "nearness". We extend this philosophical application of the simplex to explore philosophical structures in the sense of universal algebra.

3. Philosophical Abstract Simplex

In 2011, B. Sims applied abstract simplicial structures to philosophical structures to mathematically describe philosophical kinship among people having the same philosophy [10].

In general, the cognitive philosophical space is not a metric space. In the context of model theory, the cognitive philosophical space is a generalized structure $\mathcal{A} = (A, \boldsymbol{\sigma}, \boldsymbol{I})$, where A is the set of symbols (alphabets) of the structure, $\boldsymbol{\sigma}$ is the set of mental "grammar" rules, and $\boldsymbol{I}$ is an interpretation function bearing semantics for A and $\boldsymbol{\sigma}$. These generalized simplexes can be topological in nature, do not need a coordinate system, and suffice to describe cognitive structures. Also note that any philosophy/theory, as a simplicial structure, has no geometric origin, so it resembles an affine cognitive structure. However, in this work we make use of the geometric representation of the simplex as a pedagogical tool for our investigations.

Definition 3. (A Philosophy) We consider a philosophy or theory $\mathcal{A}$ as a generalized abstract simplicial structure *spanned* by an *independent set* of principles or axioms (vertices), $P = \{a_1, a_2, \dots a_n\}$:

(i). Span: Every statement $x \in \mathcal{A}$ in the philosophy is some "combination" of its principles, P. Here "combination" means formula or statement $\phi(a_1, a_2, \dots a_n)$ in terms of the spanning principles $P = \{a_1, a_2, \dots a_n\}$. So that if $x \in \mathcal{A}$ then $x = \phi(a_1, a_2, \dots a_n)$.

(ii). Independent set of principles: We say that $P = \{a_1, a_2, \dots a_n\}$ is an independent set of principles if no to principles each share the same definition, and no principle a_i is defined in terms of the other principles; that is, $a_i \neq \phi(a_1, a_2, \dots a_n)$.

Definition 3.3. Given a philosophy or theory $\mathcal{A}$ with an independent set of principles $P = \{a_1, a_2, \dots a_n\}$, the philosophy/theory is the set of all formulas ϕ in terms of P that satisfy $\mathcal{A}$, written

$$\mathcal{A} = \left\{\phi(a_1, a_2, \dots a_n) \middle| \mathcal{A} \vDash \phi \text{ and } a_j \in P\right\},$$

where $\vDash$ means satisfaction; that is, "formula ϕ satisfies the philosophy $\mathcal{A}$". Proper subsets of the principles span to create the "faces" of the philosophical simplex. Here, a philosophical "face" means a philosophical statement space that is some combination of a proper subset of the total principles.

Definition 4. (Substructure) The formula space $\boldsymbol{B_k}$ is a substructure (face) of a philosophy/theory $\mathcal{A} = span\{a_1, a_2, \dots a_n\}$ if

$$\boldsymbol{B_k} = \{\phi(a_1, a_2, \dots a_k) | \mathcal{A} \vDash \phi \text{ and } k < n\},$$

written, $\boldsymbol{B_k} < \mathcal{A}$; that is, $\boldsymbol{B_k}$ is a substructure of the philosophy/theory $\mathcal{A}$, since for every formula $\phi \in \boldsymbol{B_k}$, $\mathcal{A} \vDash \phi$.

Here, a formula $\phi(a_1, a_2, \dots a_n)$ is not merely a logic formula, we expect that there exists a mental sentiment space from which the person "holding" the philosophy $\mathcal{A}$**,** associates a sentiment λ_j , or not, to its respective principle a_j.

4. General Gluing Construction

In a Set Categorical setting, gluing is the act of identifying and combining certain sets of a topological space together. Consider sets X, Y, Z and functions $f: X \to Z$ and $g: Y \to Z$, in Figure 1, so that whenever $f(x_n) = z_i$ and $g(y_m) = z_i$ for $x_n \in X$, $y_m \in Y$, and $z_i \in Z$, we say x_n and y_m are "related", written $x_n \sim y_m$.

Thus, specific outputs of the functions f and g define an equivalence relation, $\sim$, on the disjoint union of sets X, Y written, $X \coprod Y$. Elements $x_n \in X$, $y_m \in Y$ are equivalent and can be glued if and only if $f(x_n) = g(y_m)$.

Also, under the equivalence relation, all elements in $f^{-1}(z_k) \subset X$ are glued, since for all $x_n, x_p \in f^{-1}(z_k)$, $f(x_n) = z_k = f(x_p)$ so that $x_n \sim x_p$; $f^{-1}(z_k)$ is an equivalence class. A similar statement is true for gluing with respect to $g^{-1}(z_j) \subset Y$. Consequently, if $f(x_n) = z_i$ and $g(y_m) = z_i$ then we have $x_n \sim y_m$ in $X \coprod Y$.

Now, $X \coprod Y$ is not a typical disjoint union. Under the equivalence relation, $\sim$, it can now be used to construct a "glued" structure composed of X and Y glued at the "seam", $\boldsymbol{K}$, where

$$\boldsymbol{K} = \{(x, y) \in X \times Y | \ f(x) = z_i = g(y), \text{and } \ z_i \in Z\}.$$

Define $A \subset X$ to be $A = \{x|\ f(x) = g(y)\ for\ some\ y \in Y\}$, and $B \subset Y$ to be $B = \{y|\ g(y) = f(x)\ for\ some\ x \in X\}$.

Now, joining X and Y by the equivalence relation $\sim$, the glued union $X \cup_{\sim} Y$ can be written in the form of a union of disjoint sets

$$X \cup_{\sim} Y = K \cup \{x | x \in (X - A)\} \cup \{y |\ y \in (Y - B)\}. \qquad (1)$$

We note that the equivalence classes $f^{-1}(z) \subset A$ and $g^{-1}(z) \subset B$ if and only if there exists $(x, y) \in X \times Y$ such that $f(x) = g(y) = z$. Subsets A and B can be written as the union of equivalence classes

$A = \cup_{z \in Z}\{f^{-1}(z) \mid f(x) = g(y) = z\ for\ some\ y \in Y\}$ and

$B = \cup_{z \in Z}\{g^{-1}(z) \mid f(x) = g(y) = z\ for\ some\ x \in X\}$.

Given the functions $f: X \to Z$ and $: Y \to Z$, we can define maps into the glued structure $h: Z \to X \cup_{\sim} Y$, and equivalence class maps $c_X: X \to X \cup_{\sim} Y$ and $c_Y: Y \to X \cup_{\sim} Y$, such that the diagram in Figure 2 commutes.

Given the functions $f: X \to Z$ and $: Y \to Z$, denote $[x]_z = f^{-1}(z)$ and $[y]_z = g^{-1}(z)$, then define equivalence class maps $c_X: X \to X \cup_{\sim} Y$ and $c_Y: Y \to X \cup_{\sim} Y$, to be

$$c_X([x]_z) = \begin{cases} [x]_z, if\ [x]_z \subset X - A \\ ([x]_z, [y]_z), if\ [x]_z \subset A \end{cases}, \text{ and} \qquad (2)$$

$$c_Y([y]_z) = \begin{cases} [y]_z, if\ [y]_z \subset Y - B \\ ([x]_z, [y]_z), if\ [y]_z \subset B \end{cases}.$$

Define $Z_K \subset Z$, to be $Z_K = \{z|\ f(x) = g(y) = z\ for\ some\ (x, y) \in X \times Y\}$, define the map $h: Z \to X \cup_{\sim} Y$ by

$$h(z) = \begin{cases} [x]_z, z \in (Z - Z_K) and\ f(x) = z \\ [y]_z, z \in (Z - Z_K)\ and\ g(y) = z\ . \\ ([x]_z, [y]_z), z \in Z_K \end{cases} \qquad (3)$$

Now the diagram in Figure 1 commutes; that is, $c_X = hf$ and $c_Y = hg$.

The construction in Figure 1 contains a categorical Co-product of X and Y, where the triple (Z, f, g) is the co-product with maps $f: X \to Z$ and $g: Y \to Z$, where the map h is uniquely defined in terms of equivalence classes $[x]_z = f^{-1}(z)$ and $[y]_z = g^{-1}(z)$.

Also, for all $z \in Z_K$ the following functions agree, $h(z) = c_X f^{-1}(z) = c_Y g^{-1}(z)$. For each $z \in Z_K$, the equivalence class coordinate $([x]_z, [y]_z)$, in Equations (2) and (3), is a subset of $\boldsymbol{K}$ and can be written as

$$(x, y)_z = \{(x, y) \in A \times B |\ f(x) = g(y) = z,\ for\ z \in Z_K\}.$$

For h restricted to Z_K, $h = \{(z, ([x]_z, [y]_z)) |\ f(x) = g(y) = z, for\ \ z \in Z_K\}$. Thus, there is a projection map π from h onto $\boldsymbol{K}$, $\pi_z : h \to \boldsymbol{K}$, given by

$$\pi_z(z, ([x]_z, [y]_z)) = ([x]_z, [y]_z) \subset \boldsymbol{K}. \qquad (4)$$

The map composition $\pi_z h = \boldsymbol{K}$, where $\boldsymbol{K} = \bigcup_{z \in Z_K} [x, y]_z$, in terms of the union of equivalence class coordinates. The set $\boldsymbol{K}$ defines a bijective equivalence class map ρ from A to B, $\rho: A \to B$, defined by

$$\rho([x]_z) = [y]_z. \qquad (5)$$

Our map ρ will be important for discussion on gluing criteria and, semantic and structure preserving maps between philosophies/theories.

5. Gluing Philosophies

We develop criteria for gluing philosophies, where the philosophies are objects in a category of generalized algebras- a cognitive substructure of the mind. The morphisms in this "mental" category are algebra homomorphisms and *statement maps*. Consider two different philosophies $\boldsymbol{X}_{\boldsymbol{P}_1}$ and $\boldsymbol{Y}_{\boldsymbol{P}_2}$ spanned by principles/axioms $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, respectively, as in Definition 3.3.
Say $\boldsymbol{P}_1 = \{a_1, a_2, \ldots a_n\}$ and $\boldsymbol{P}_2 = \{b_1, b_2, \ldots b_m\}$. *Examine* words or statements in both sets $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, to *determine* which words and statements in principle set $\boldsymbol{P}_1$ share the **same meaning** with words and statements in principle set $\boldsymbol{P}_2$, creating an **equivalence** relation between subsets of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$.

Define subsets $\boldsymbol{A} \subset \boldsymbol{P}_1$ and $\boldsymbol{B} \subset \boldsymbol{P}_2$ to be
$\boldsymbol{A} = \{words\ a_i \in \boldsymbol{P}_1 |\ a_i\ has\ the\ same\ meaning\ as\ some\ b_i \in \boldsymbol{P}_2\}$, and
$\boldsymbol{B} = \{words\ b_i \in \boldsymbol{P}_2 |\ b_i\ has\ the\ same\ meaning\ as\ some\ a_i \in \boldsymbol{P}_1\}$.
Each $a_i \in \boldsymbol{P}_1$ is distinct in meaning, as in definition 3, and similar for the $b_i \in \boldsymbol{P}_2$. We let the words a_i and b_i have the same index when they have the same meaning, say z_i , in a definition space $\boldsymbol{Z}$. Also, the cardinality of sets $\boldsymbol{A}$ and $\boldsymbol{B}$ will be the same.

In general, not all words in $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ will share the same meaning. But if there are some, $\boldsymbol{A}$ and $\boldsymbol{B}$ will be proper subsets of $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, respectively. Let $\boldsymbol{S}_A = \boldsymbol{span\ A}$ and $\boldsymbol{S}_B = \boldsymbol{span\ B}$. By Definition 4, $\boldsymbol{S}_A$ and $\boldsymbol{S}_B$ are substructures (faces) of $\boldsymbol{X}_{\boldsymbol{P}_1}$ and $\boldsymbol{Y}_{\boldsymbol{P}_2}$, respectively; written $\boldsymbol{S}_A \prec \boldsymbol{X}_{\boldsymbol{P}_1}$ and $\boldsymbol{S}_B \prec \boldsymbol{Y}_{\boldsymbol{P}_2}$.

By defining substructures, $\boldsymbol{S_A} \prec \boldsymbol{X_{P_1}}$ and $\boldsymbol{S_B} \prec \boldsymbol{Y_{P_2}}$, to be faces spanned by equivalent principles, $a_i \sim b_i$, of our original philosophies $\boldsymbol{X_{P_1}}$ and $\boldsymbol{Y_{P_2}}$, we have identified philosophical substructures for gluing.

It is necessary but not sufficient that there is a one-to-one match between subsets of principles $\boldsymbol{A}$ and $\boldsymbol{B}$. The way principles are composed into formulas satisfied in one philosophy, $\boldsymbol{X_{P_1}}$, must be properly "communicated" in formula composition and satisfied in the target philosophy $\boldsymbol{Y_{P_2}}$, to be glued. Thus, a formula map from substructure $\boldsymbol{S_A} \prec \boldsymbol{X_{P_1}}$ to substructure $\boldsymbol{S_B} \prec \boldsymbol{Y_{P_2}}$ must also preserve semantical satisfaction across structure "grammars", $\boldsymbol{\sigma_X}$ and $\boldsymbol{\sigma_Y}$, in-order for an articulation between the two substructures to be valid. For the satisfaction relation, $\models$, to hold in both philosophies, a map $\tilde{\rho}: \boldsymbol{S_A} \rightarrow \boldsymbol{S_B}$ must be a generalized formula homomorphism, taking formulas satisfied in the "grammar" $\boldsymbol{\sigma_X}$ of substructure $\boldsymbol{S_A}$ to formulas satisfied in the "grammar" $\boldsymbol{\sigma_Y}$ of substructure $\boldsymbol{S_B}$.

Definition 5. (Homomorphism)

Given two structures $\mathcal{X}$ and $\mathcal{Y}$, spanned by sets $X = \{a_1, a_2, a_3, \ldots a_t\}$ and $Y = \{b_1, b_2, b_3, \ldots b_w\}$, respectively; a homomorphism from X to Y is a map $\rho: X \rightarrow Y$ such that whenever there is a formula $\Phi_A(a_i)$ in $\mathcal{X}$ with, $\mathcal{X} \models \Phi_A(a_1, a_2, a_3, \ldots a_n)$, there exist a formula $\Phi_B(b_i)$ in $\mathcal{Y}$ where

$$\mathcal{Y} \models \Phi_B(\rho(a_1), \rho(a_2), \rho(a_3), \ldots, \rho(a_n), b_{n+1}, \ldots, b_m),$$

for $a_i \in X$, $b_i = \rho(a_i)$, and $i = 1,2,3, \ldots, n$.

The homomorphism ρ in Definition 5 is a bijective map from $\{a_1, a_2, a_3, \ldots a_n\} \subset X$ to $\{b_1, b_2, b_3, \ldots b_n\} \subset Y$. Also, in this most general sense of homomorphism, it is not necessary that $n = m$, in the respective formulas $\Phi_A(a_i) \in \mathcal{X}$ and $\Phi_B(b_i) \in \mathcal{Y}$, since the language of $\mathcal{Y}$ may require additional principles $\{b_{n+1}, \ldots, b_m\}$ to logically support $\{b_1, b_2, b_3, \ldots b_n\}$ in the grammar of $\mathcal{Y}$, so that $\Phi_B(\rho(a_1), \rho(a_2), \rho(a_3), \ldots, \rho(a_n), b_{n+1}, \ldots, b_m)$ is satisfied in $\mathcal{Y}$.

Definition 6. (Satisfaction Equivalence)

The formulas, $\Phi_A(a_i) \in \mathcal{X}$ and $\Phi_B(b_i) \in \mathcal{Y}$, are *satisfaction equivalent with respect to* $\models$ if and only if there exists a homomorphism $\rho: X \rightarrow Y$, between their spanning sets, such that $\mathcal{X} \models \Phi_A(a_i)$ if and only if $\mathcal{Y} \models \Phi_B(b_i)$ where $b_i = \rho(a_i)$; written, $\Phi_A(a_i) \leftrightarrow \Phi_B(b_i)$.

Proposition 1. Let, $\sim$, be an equivalence relation on the members of disjoint sets X and Y, spanning the structures $\mathcal{X}$ and $\mathcal{Y}$, respectively, given by $a_n \sim b_n$ for some $(a_n, b_n) \in X \times Y$. Then,

1. there exists an equivalence class map $\rho : X \to Y$, defined by $\rho([a_n]) = [b_n]$,

2. The Satisfaction equivalence, $\leftrightarrow$, given by $\Phi_A(a_i) \leftrightarrow \Phi_B(b_i)$ for $(\Phi_A(a_i), \Phi_B(b_i)) \in \mathcal{X} \times \mathcal{Y}$, is an equivalence relation induced by $\sim$, via ρ.

3. (Statement map) The bijective formula map $\tilde{\rho}$ from substructure $\boldsymbol{span}\{a_i\}$ to $\boldsymbol{span}\{b_i\}$, defined by $\tilde{\rho}(\Phi_A(a_i)) = \Phi_B(b_i)$ if and only if $\Phi_A(a_i) \leftrightarrow \Phi_B(b_i)$, is a *statement map* induced by $\sim$.

Proposition 2. (Inheritance) Given two structures $\mathcal{X}$ and $\mathcal{Y}$, spanned by sets $X = \{a_1, a_2, a_3, \ldots a_t\}$ and $Y = \{b_1, b_2, b_3, \ldots b_w\}$, respectively, with a homomorphism from X to Y, $\rho : X \to Y$, then the gluing of the structures $\mathcal{X}$ and $\mathcal{Y}$, $\mathcal{X} \cup_{\tilde{\rho}} \mathcal{Y}$, via the bijective statement map $\tilde{\rho}$, is *inherited* from the gluing of sets X and Y, $X \cup_{\sim} Y$, via the equivalence relation $\sim$.

Gluing Criteria:
Two or more philosophies can be "glued" if they have
1. An equivalence relation on subsets of words: have equivalent words/phrases/sentences, e.g., words/phrases/sentences that have the same meaning, or represent the same concept or thing.

2. There exists a statement homomorphism between their substructures, where the substructures are spanned by their respective subsets of equivalent words/phrases/statements.

6. Examples of Gluing Philosophies.

Example 1. Consider some definitive issues from the set of the Republican and Democratic 2016 Platforms, listed in Table 1 [11,12]. The issue lists, here, are not exhaustive; nor do the lists infer any political-test on what is or is not "Republican" or "Democratic". The principles in the political case are some issues that candidates campaign on, and are only used here to explicate the process and implication of gluing philosophies.

We say that issue $\boldsymbol{a_n}$ is **equivalent** to issue $\boldsymbol{b_n}$ when they are *defined the same and will have the same intended result when applied*, written $a_n \sim b_n$. Equivalent issues are in red, Table 1.

Let $\mathcal{T}_R$ be the Republican ideology spanned by its principle set $R = \{a_1, a_2, a_3, \ldots, a_{14}\}$ and $\mathcal{T}_D$ be the Democratic ideology spanned by its principle set $D = \{b_1, b_2, b_3, \ldots, b_{14}\}$, Definition 3.3.

The equivalent issues are $a_2 \sim b_2$, $a_3 \sim b_3$, $a_4 \sim b_4$, $a_5 \sim b_5$, $a_7 \sim b_7$, and $a_9 \sim b_9$. Let subset $A = \{a_2, a_3, a_4, a_5, a_7, a_9\}$ and subset $B = \{b_2, b_3, b_4, b_5, b_7, b_9\}$.

Consider that we can find some set Z of definitions (dictionary), in "common language". How the two ideologies, Republican and Democratic, theoretically satisfy the requirements for the gluing construction in Section 3 as follows.

Let $Z_K \subset Z$ be such that a_n and b_n map to the same $z_n \in Z_K$: "*meaning and intended result when applied*", for $n = 2,3,4,5,7, and\ 9$. There exists functions $f:R \rightarrow Z$ and $g: D \rightarrow Z$ such that $f(a_n) = z_n$ and $g(b_n) = z_n$, for $n = 2,3,4,5,7, and\ 9$. The triple (Z, f, g) is the co-product of R and D, where Z is a "dictionary".

Let $\boldsymbol{K} = \{(a_n, b_n) \in R \times D | f(a_n) = g(b_n) = z_n, and\ \ z_n \in Z\}$.

Thus, for the Political issues, we have equivalence class maps $c_R: R \rightarrow R \cup_\sim D$ and $c_D: D \rightarrow R \cup_\sim D$, and $h: Z \rightarrow R \cup_\sim D$ defined in Equations 2 and 3, respectively. The set $R \cup_\sim D$ is only a glued set of political issues, glued along set $\boldsymbol{K}$.

Intrinsic to the system of equivalence class maps, there exists the projection map $\pi_z: h \rightarrow \boldsymbol{K}$ defined in equation 4, from which is derived the bijective equivalence class map ρ from A to B, $\rho: A \rightarrow B$, defined in equation 5, $\rho([a_n]_z) = [b_n]_z$.

From the above equivalences, issues in subset A and subset B are in a one-to-one correspondence through the bijective map $\rho: A \longrightarrow B$ defined by $\rho(a_n) = b_n$ for $n = 2,3,4,5,7, and\ 9$. The map ρ is the equivalence class map described in Equation (5), where in this example, each of the equivalence classes $[a_n]$ and $[b_n]$ are singleton sets.

Also, since the coordinates of the map ρ are strictly identified by issues that are "*defined the same and will have the same intended result when applied*", ρ preserves semantics.

Now arguments ϕ_A^j that satisfy the Republican ideology,
$\mathcal{T}_R \vDash \phi_A^j(a_2, a_3, a_4, a_5, a_7, a_9)$, will also satisfy arguments ϕ_B^j in the Democratic ideology where,

$$\mathcal{T}_D \vDash \phi_B^j(\rho(a_2), \rho(a_3), \rho(a_4), \rho(a_5), \rho(a_7), \rho(a_9)).$$

Now by Proposition 1, ρ is the homomorphism defined in Definition 5. $\phi_A^j(a_n)$ and $\phi_B^j(b_n)$ are ***equivalent arguments with respect to semantic satisfaction*** $\vDash$, for $n = 2,3,4,5,7, and\ 9$; and since $S_A \prec \mathcal{T}_R$ and $S_B \prec \mathcal{T}_D$, there exists a bijective statement map $\tilde{\rho}: S_A \longrightarrow S_B$, such that $\tilde{\rho}\left(\phi_A^j(a_n)\right) = \phi_B^j(b_n)$.

The structures S_A and S_B need not be "complete" sub-theories themselves; they only need to be faces defined as in Definition 4 to qualify for gluing. The two ideologies, Republican $\mathcal{T}_R$ and Democratic $\mathcal{T}_D$ can be "glued" at their respective substructures S_A and S_B, via the map $\tilde{\rho}$. By Proposition 2, the gluing of the ideologies $\mathcal{T}_R$ and $\mathcal{T}_D$ via homomorphism $\tilde{\rho}$ is inherited from the gluing of political issue sets R and D via the equivalence relation $\sim$. That is, the glued ideology $\mathcal{T}_R \cup_{\tilde{\rho}} \mathcal{T}_D$ is inherited from the glued issue set $R \cup_{\sim} D$, and is explicitly written in terms of Definition 3.3 by

$$\mathcal{T}_R \cup_{\tilde{\rho}} \mathcal{T}_D = \mathcal{K} \cup \{\phi(a_n) | \mathcal{T}_R \vDash \phi,\ a_n \in (\mathrm{R} - \mathrm{A})\} \cup \{\phi(b_n) | \mathcal{T}_D \vDash \phi,\ b_n \in (\mathrm{D} - \mathrm{B})\},$$

where the "seam" $\mathcal{K}$, or "common ground" is the set of coordinate equivalent arguments:

$$\mathcal{K} = \{(\phi(a_n), \phi(b_n)) |\ \mathcal{T}_R \times \mathcal{T}_D \vDash (\phi(a_n), \phi(b_n)),\ (a_n, b_n) \in A \times B, and\ b_n = \rho(a_n)\}.$$

In general, the statement map $\tilde{\rho}: S_A \to S_B$ presented here, is a function for mapping interpretations from the substructure S_A of ideology $\mathcal{T}_R$ onto substructure S_B of the target ideology $\mathcal{T}_R$, preserving semantics and grammar.

Example 2. Consider some definitive beliefs from the general Christian and Islamic theology, listed in Table 2 [13,14]. Again, as in example 1, the beliefs lists, here, are not exhaustive; nor do the lists infer any religious-test on what is or is not "Christian" or "Islamic". The principles in the theological case are some beliefs that can be found in each of the two religions, and are used here to explicate the process and implication of gluing philosophies.

From table 2. Let $\mathcal{T}_J$ be the Christian theology spanned by its principle set $\mathcal{J} = \{a_1, a_2, a_3, \dots, a_{12}\}$ and $\mathcal{T}_I$ be the Islamic theology spanned by its principle set $I = \{b_1, b_2, b_3, \dots, b_{12}\}$, Definition 3.3. The equivalent beliefs are $a_1 \sim b_1$, $a_3 \sim b_3$, $a_6 \sim b_6$, $a_9 \sim b_9$, $a_{10} \sim b_{10}$, and $a_{12} \sim b_{12}$.

As in example 1, we consider some common dictionary, set Z of definitions, giving rise to a co-product, triple (Z, f, g), of the Christian and Islamic theologies. Applying the gluing construction from Section 3, the glued set of beliefs $\mathcal{J} \cup_{\sim} I$ follows from Equations 2 and 3, similar to Example 1. So, we focus here on the theological structure $\mathcal{T}_J \cup_{\tilde{\rho}} \mathcal{T}_I$ glued via its semantic preserving homomorphism $\tilde{\rho}$.

Let S_A be the Christian sub-structure spanned by subset $A = \{a_1, a_3, a_6, a_9, a_{10}, a_{12}\}$ and S_B be the Islamic sub-structure spanned by subset $B = \{b_1, b_3, b_6, b_9, b_{10}, b_{12}\}$. With a semantic preserving homomorphism $\rho: A \longrightarrow B$ defined by $\rho(a_n) = b_n$ for $n = 1,3,6,9,10, and\ 12$, the Christian and Islamic theologies can be glued at their respective theological substructures $S_A \prec \mathcal{T}_J$ and $S_B \prec \mathcal{T}_I$, since for every formula or argument $\phi_A^j(a_n) \in S_A$ that satisfies the Christian theology, $\mathcal{T}_J \vDash \phi_A^j(a_1, a_3, a_6, a_9, a_{10}, a_{12})$, there will exist an argument $\phi_B^j(b_n) \in S_B$ in the Islamic theology where,

$$\mathcal{T}_I \vDash \phi_B^j(\rho(a_1), \rho(a_3), \rho(a_6), \rho(a_9), \rho(a_{10}), \rho(a_{12})),$$

so that $\phi_A^j(a_n)$ and $\phi_B^j(b_n)$ are ***equivalent arguments with respect to semantic satisfaction*** $\vDash$, for $n = 1,3,6,9,10, and\ 12$. By Proposition 1 there exists a bijective statement formula map $\tilde{\rho}: S_A \longrightarrow S_B$, such that $\tilde{\rho}\left(\phi_A^j(a_n)\right) = \phi_B^j(b_n)$.

By Proposition 2, the glued theological structure is

$$\mathcal{T}_J \cup_{\tilde{\rho}} \mathcal{T}_I = \mathcal{K} \cup \{\phi(a_n) | \mathcal{T}_J \vDash \phi,\ a_n \in (J - A)\} \cup \{\phi(b_n) | \mathcal{T}_I \vDash \phi,\ b_n \in (I - B)\},$$

where the "seam" $\mathcal{K}$, or "common ground" is the set of coordinate equivalent arguments:

$$\mathcal{K} = \{(\phi(a_n), \phi(b_n)) |\ \mathcal{T}_J \times \mathcal{T}_I \vDash (\phi(a_n), \phi(b_n)),\ (a_n, b_n) \in A \times B, and\ b_n = \rho(a_n)\}.$$

Again, our homomorphism $\tilde{\rho}$ is a function for mapping interpretations from substructure S_A of theology $\mathcal{T}_J$ onto the substructure S_B of the target theology $\mathcal{T}_I$.

7. Universal Property

Definition 6. (Universal Property)
Let M and X_i be objects with map $f_i: X \to M$, for $i = 1,2,3 \dots n$. The object M is said to have the *universal property* with respect to (X_i, f_i), if for any other object N with maps $g_i: X \to N$, there exists a *unique* map $u: M \to N$ such that $g_i = uf_i$, for $i = 1,2,3 \dots n$. That is the diagram in Figure 2 commutes for $i = 1,2,3 \dots n$.

Two main points from Definition 6; (1) the object M is "general" enough to take on maps f_i from X, and (2) M is "specific" enough to admit a unique map u such that the map g_i "factors through" the pair (M, u) given by $g_i = uf_i$, for $i = 1,2,3 \dots n$.

Proposition 3. (Universal Property of the Gluing Topology II)
If $X \cup_\sim Y$ is a glued set constructed by gluing sets X and Y under an equivalence relation $\sim$, then $X \cup_\sim Y$ has the universal property with respect to X and Y [2].

Social interpretation of Universal Property
Let G_{P_1} and G_{P_2} be two communities of people with philosophies, ideologies, or religions spanned by sets of principles P_1 and P_2, respectively. Assume that their philosophical (ideology or religion) structures $\mathcal{T}_{P_1}$ and $\mathcal{T}_{P_2}$ satisfy the gluing criteria and are glued, producing $\mathcal{T}_{P_1} \cup_{\tilde{\rho}} \mathcal{T}_{P_2}$. Let $c_{P_1}: \mathcal{T}_{P_1} \to \mathcal{T}_{P_1} \cup_{\tilde{\rho}} \mathcal{T}_{P_2}$ and

$c_{P_2}: \mathcal{T}_{P_2} \to \mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2}$ be generalized interpretation maps into $\mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2}$, defined by the people in the respective communities.

If there is any common real-world problem n that must be solved, define maps $m_{P_1}: \mathcal{T}_{P_1} \to \mathrm{N}$ and $m_{P_2}: \mathcal{T}_{P_2} \to \mathrm{N}$ to be the *methodologies* that each community will implement to arrive at a solution in N, for the problem n, where N is a cognitive solution space for n. The community $\boldsymbol{G}_{P_1}$ implements methodology m_{P_1} to produce a solution $s_1 \in \mathrm{N}$, dependent on a set of formulas found in $\mathcal{T}_{P_1}$, written $m_{P_1}\{\phi^j_{P_1}\} = s_1$, for some $\phi^j_{P_1} \in \mathcal{T}_{P_1}$. Similarly, community $\boldsymbol{G}_{P_2}$ implements methodology m_{P_2} to produce solution $s_2 = m_{P_2}\{\phi^i_{P_2}\}$, for some $\phi^i_{P_2} \in \mathcal{T}_{P_2}$.

By Proposition 3, the glued philosophical structure $\mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2}$ has the Universal property, which guarantees that there exists a unique *methodology* $u: \mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2} \to N$, such that $m_{P_1} = uc_{P_1}$ and $m_{P_2} = uc_{P_2}$.

The fundamental social point here is that the unique methodology u is a common factor- common methodology- among communities $\boldsymbol{G}_{P_1}$ and $\boldsymbol{G}_{P_2}$. The unique factorization of each community's methodology/solution, through the glued philosophical structure $\mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2}$, can reduce the perception of ambiguity and facilitate a more unified problem solving approach among members in communities $\boldsymbol{G}_{P_1}$ and $\boldsymbol{G}_{P_2}$.

With respect to completeness, in model theory, it has been shown in [15] that if an abstract model $\boldsymbol{M_A}$ satisfies a universal property and simulates a concrete model $\boldsymbol{M_C}$, then the concrete model also satisfies the same universal property. In the abstract case, gluing two Algebra structures produces a new structure having a universal property. In our concrete model (human philosophical and social environment), the glued philosophical structures will also possess the same universal property.

8. Human Impedance to the Universal Property

Ability to Construct a Gluing

Theoretically, the glued philosophy, $\mathcal{T}_{P_1} \cup_{\breve{\beta}} \mathcal{T}_{P_2}$, in the social interpretation guarantees the existence of the logical implications (effects) for people or communities, $\boldsymbol{G}_{P_1}$ and $\boldsymbol{G}_{P_2}$, "factoring through" the glued structure. However, there are several challenges to creating and implementing a glued philosophical structure, due to human properties. Fundamentally, the mental capacity of a person to cognitively construct a glued philosophy is in question here. In a categorical mental setting, Phillips in [5] noted that the success level of subjects solving problems involving Transitive Inference or Class Inclusion required the subject to have the mental capacity to construct categorical Products and Co-products. For us, the existence of a glued philosophy depends on the existence of a person(s) with the capacity to construct glued structures, and their willingness to construct such a structure. Also, while the intellectual capacity may be present for an individual, there can be sentimental or social constraints that impede construction. Even if the glued philosophy is constructed by persons having the mental capacity, other people believing in their respective component philosophies my not apply or implement the glued principles according to definition.

Emotional Influences

Emotion or sentiments, to some degree, are ever present, during person to person interaction or communication. A communication, $\tilde{\rho}: \mathcal{T}_{P_1} \longrightarrow \mathcal{T}_{P_2}$, representing a homomorphic mapping of equivalent statements $\phi^k_{P_1}$ from person $A \in G_{P_1}$ to person $B \in G_{P_2}$, may not also map the sentiments of person A for such statements to person B. This could result from emotion-cognition pairing. The concept of emotion pairing with cognitions has been initiated by Izad in 1992 in a study on combining feeling and thought, through affective-cognitive structures [16]. In 2011, Sims described emotion-principle pairing in an abstract simplex, where λ_j represents a numerical measure of a person's *sentiment* for its corresponding principle a_j , and is "paired" with the principle as a generalized product $\lambda_j a_j$ [9]. In this sense, a set of statements with paired emotion-cognition, $\{\phi(\lambda_j a_j)\}^k_{j=1}$, is an abstract module over a sentiment space $\mathcal{S}$, where $\lambda_j \in \mathcal{S}$.

Example 3. Let the principles $a_n \in P_1$ and $b_n \in P_2$ be "equivalent", so that $\rho(a_n) = b_n$ and let λ_{n_A} and λ_{n_B} represent person $A's$ *sentiment* and person $B's$ *sentiment* for their respective principles a_n and b_n, where $\lambda_{n_A} \neq \lambda_{n_B}$. Now, under emotion-principle pairing, $\rho(\lambda_{n_A} a_n) \neq \lambda_{n_B} b_n$, so that it is possible that $\tilde{\rho}\big(\phi^k_{P_1}(a_1, a_2, \dots, a_j, \dots, a_k)\big) = \phi^k_{P_2}(b_1, b_2, \dots, b_n, \dots, b_k)$, while $\tilde{\rho}\big(\phi^k_{P_1}(\lambda_1 a_1, \lambda_2 a_2, \dots, \lambda_{n_A} a_j, \dots, \lambda_k a_k)\big) \neq \phi^k_{P_2}(\sigma_1 b_1, \sigma_2 b_2, \dots, \lambda_{n_B} b_n, \dots, \sigma_k b_k)$, Thus, sentiments may not map from person to person during implementation of the glued philosophy.

This challenge involves personal sentiments where; although principles a_n may logically be glued to b_n by intellectually acknowledging their common definition, the persons A and B may have such sentimental differences to the degree that the sentimental differences influence or abrogate the joint implementation of a methodology through $\mathcal{T}_{P_1} \cup_{\tilde{\rho}} \mathcal{T}_{P_2}$ whenever the method involves a_n and b_n.

Social Pressure

Social pressure can also impede the application of joint methodologies.

Example 4. In the political environment, members of the Republican or Democratic party may logically see a unique bipartisan solution u to bail-out banks and large companies by "factoring" through $\mathcal{T}_R \cup_{\tilde{\beta}} \mathcal{T}_D$; however, actual implementation of the solution u may be viewed as a "weakness" by the public constituents of those members. The anxieties over public-opinion-dependent political success or demise, can impede the bipartisan implementation of bail-out methods.

Contextual Influences

Another cognitive challenge for the two communities G_{P_1} and G_{P_2} could arise from quantum cognitive affects. Aerts (2009) applied quantum mechanics to explain the *interaction* of cognitive concepts, say H, J, where the union $H \cup J$ can be viewed as the "combination" of two concepts; or, the union can be viewed as a whole new concept with its own new context due to the cognitive quantum superposition of the contexts of H and J [17].

When faced with the decision to implement methodologies based on the glued philosophy $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$, some individuals from communities G_{P_1} or G_{P_2} may agree on implementation of a methodology to handle an issue n, because they agree that n is satisfied in the context of their philosophy $\mathcal{T}_{P_1}$ or $\mathcal{T}_{P_2}$, a decision made without the contextual influences of $\mathcal{T}_{P_1}$ and $\mathcal{T}_{P_2}$; however, if the glued philosophy $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$ is perceived by the communities to be a "wholly new concept", then the presence of cognitive quantum interference means that the *contexts* of philosophies $\mathcal{T}_{P_1}$ and $\mathcal{T}_{P_2}$ cognitively *interact* with each other to influence decision making through the glued philosophy $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$. Because of this cognitive quantum interference, a newly perceived context under $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$ emerges for those individuals. Perceiving $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$ as a "whole", issue n is now subject to the new context, without exclusive cognitive reference to $\mathcal{T}_{P_1}$ or $\mathcal{T}_{P_2}$. Cognitive quantum effects can impede joint implementation of methodologies.

Example 5. Both political parties, Democrat and Republican, support capitalism under a "Free Market Economy". While competition is one of capitalisms key ingredients, there are those in society (constituents of both parties) that are considered to be competitively disadvantaged. While there could be a unique bipartisan method implemented by both parties factoring through $\mathcal{T}_R \cup_{\tilde{\beta}} \mathcal{T}_D$, to solve the *competitively disadvantaged* issue n; Welfare, Education, and Affirmative Action policies (methods) put forth to solve the issue n, again, may not find favorable bipartisan implementation via $\mathcal{T}_R \cup_{\tilde{\beta}} \mathcal{T}_D$, if the ideology $\mathcal{T}_R \cup_{\tilde{\beta}} \mathcal{T}_D$ takes on a "Communist" or "Socialist" context, by the cognitive quantum effect; even though both $\mathcal{T}_R$ and $\mathcal{T}_D$, each, have a "Free Market Economy" context with respect to the competitively disadvantaged.

Example 6. Based on the life of Jesus and Muhammad, an interfaith methodology to care for the "needy" (food, clothing, shelter, or education) could be implemented by both Christian and Muslim communities. However, by cognitive quantum effects the glued structure $\boldsymbol{R} = \boldsymbol{\mathcal{T}}_C \cup_{\tilde{\beta}} \boldsymbol{\mathcal{T}}_I$ might be viewed as a whole new religion to either one, or both communities, where the "divinity of Christ" or the "prophet-hood of Muhammad" may be minimized in the context of $\boldsymbol{R}$. In either case the joint implementation of an interfaith methodology could be impeded by members of either community.

The Homomorphism Problem

We were able to develop criteria for "gluing" different philosophies together, given that a homomorphism exists between the respective philosophies. Matching terms from different philosophies that carry the same principle is a tractable task, since we only need to learn the meaning of words, statements, terms, or symbols and find the underlying principle. The problem is to determine that the structure of every formula, ϕ_A , over (words, statements, terms, or symbols) satisfied in, say, philosophy $\boldsymbol{A}$ is preserved in a formula ϕ_B satisfied in Philosophy $\boldsymbol{B}$. This is typically termed the homomorphism problem

Figures and Tables

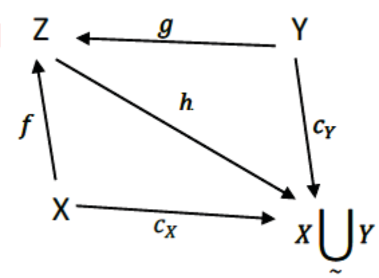

Figure 1. Illustration of gluing construction $X \cup_{\sim} Y$ in a category of sets.

Republican Issues	Democratic Issues
a_1: Inflexible Constitution	b_1: Constitution
a_2: Free Market Economy	b_2: Free Market Economy
a_3: Domestic Jobs Creation	b_3: Domestic Jobs Creation
a_4: Foreign Policy	b_4: Foreign Policy
a_5: Education (Academic Excellence)	b_5: Education (Innovation Agenda)
a_6: Health Care (Free Market)	b_6: Health Care (Universal)
a_7: Anti-Discrimination	b_7: Anti-Discrimination
a_8: Same-Sex Marriage (Against)	b_8: Same-Sex Marriage (Supports)
a_9: Restore American Families	b_9: Support Working Families
a_{10}: Environment (Deny anthropogenic cause of climate change. Supports Fossil Fuel)	b_{10}: Environment (Supports anthropogenic cause of climate change, and Clean energy transition)
a_{11}: Criminal Justice (reduce creation of new crime laws, increase restorative justice programs for victims and offenders)	b_{11}: Criminal Justice (expanding reentry programs, Federal Policing local Law Enforcement)
a_{12}: Immigration (Deport Illegal Aliens)	b_{12}: Immigration (Pathway to Citizenship for some Illegal Aliens)
a_{13}: Welfare with Transition to Independence	b_{13}: Welfare (Increase Federal Assistance)
a_{14}: Gun Control	b_{14}: Gun Control

Table 1. A list of some definitive issues from the Republican and Democratic 2016 Platforms. Equivalent issues are in red.

Christian Doctrine	Islamic Doctrine
a_1: One God	b_1: One God
a_2: The Trinity	b_2: Belief in all God's Prophets, Angels and Revelations
a_3: The Holy Sprit	b_3: The Holy Spirit
b_4: Follow Spiritual Tradition of Jesus Christ	b_4: Follow Tradition of Prophet Muhammad
a_5: Jesus Christ Son of God	b_5: Prophet-hood of Jesus the Messiah
a_6: Jesus Christ's Virgin Birth through Mary by God	b_6: Jesus the Messiah Virgin Birth through Mary by Will of God
a_7: Christ's Death, Resurrection, and Return	b_7: Jesus the Messiah and Reformer Return-last days
a_8: Salvation through Belief in Christ's Death and Resurrection	b_8: Salvation through Submission to God
a_9: Human Nature- divine but fallible	b_9: Human Nature- divine but fallible
a_{10}: Day of Judgement	a_{10}: Day of Judgement
a_{11}: Sacraments (rites and sacred objects)	b_{11}: Sacraments (Obligatory prayer and fasting)
a_{12}: Equality of Human Family	a_{12}: Equality of Human Family

Table 2. A list of some definitive beliefs that can be found in Christian and Islamic theology. Equivalent beliefs are in red.

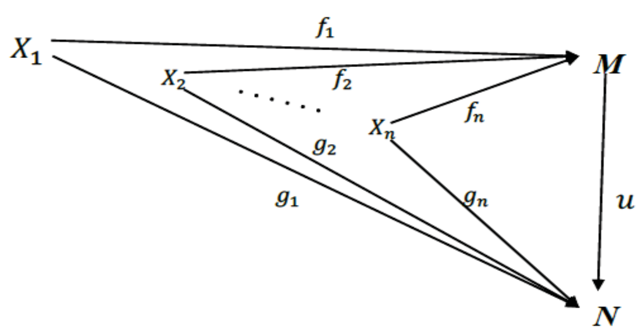

Figure 2. Maps "factoring" through *universal* object M.

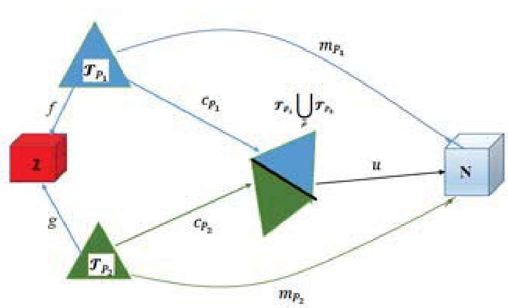

Figure 3. Illustration of universal property for cognitive sub-structures. The philosophical structure $\mathcal{T}_{P_1} \bigcup_{Z} \mathcal{T}_{P_2}$ constructed by philosophies $\mathcal{T}_{P_1}$ (blue) and $\mathcal{T}_{P_2}$ (green), glued along common face (in black). **Z** is a space of common definitions with maps from philosophies $\mathcal{T}_{P_1}$ and $\mathcal{T}_{P_2}$. N is

some solution space. Methodologies m_{P_1} and m_{P_2} factor through $\mathcal{T}_{P_1} \cup_{\tilde{\beta}} \mathcal{T}_{P_2}$ by $m_{P_1} = u \circ c_{P_1}$ and $m_{P_2} = u \circ c_{P_2}$ with unique mutual methodology u.

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References:

[1] M. Eysenck and M. Keane. Cognitive Psychology: Student's Handbook. pgs. 266- 272. Psychology Press Ltd, 2000.

[2] E. Riehl. *ON THE CONSTRUCTION OF NEW TOPOLOGICAL SPACES FROM EXISTING ONES*. semanticscholar.org, 2015.

[3] S. Hasen. CW Complexes. www.math.ksu.edu/~hansen/CWcomplexes.pdf. Retrieved April 2018

[4] M. D'Anna, C. A. Finocchiaro, and M. Fontana. Amalgamated algebras along an ideal. Commutative Algebras and Applications. Cornell University Library. arXiv:0901.1742. 2009

[5] S. Phillips and K. Wilson. Categorial Compositionality: A Category Theory Explanation for the Systematicity of Human Cognition. H. K. Friston (editor). PLoS Computational Biology. Volume 6, Issue 7, e1000858. 2010.

[6] G. Boole. An Investigation of the Laws of Thought. Cambridge Macmillan and Co. London. Chapters 1 and 2. 1854.

[7] P.J. Hilton and S. Wylie. Homology Theory, An Introduction to Algebraic Topology. Cambridge University Press, 1967.

[8] K.F Kee, L. Sparks, D.C. Struppa, and M Mannucci. Social Groups, Social Media, and Higher Dimensional Social Structures: A Simplicial Model of Social Aggregation for Computational Communication Research. Communication Quarterly (2013), 61:1, pp 35-58, 2013.

[9] J. Legrand. How far can Q-ˇanalysis go into social systems understanding? Fifth European Systems Science Congress, 2002.

[10] B. Sims. An Anthropomorphic Perspective in Homology Theory. Journal of Applied Global Research, Vol. 4, 10: 29 – 40, 2011.

[11] Republican Platform 2016. www.gopconvention2016.com. Retrieved April 2018.

[12] 2016 Democratic Party Platform. www.democrats.org/party-platform. Retrieved April 2018

[13] A. McGrath. Christian Theology: An Introduction pgs. 173 – 381. John Wyley and Sons LTD, 2017

[14] M. M. Ali. The Religion of Islam. Ahmadiyya Anjuman Ishaat Islam Lahore USA. Pgs. 102-199.

[15] P. Godefroid and R. Jagadeesan. Automatic abstraction using generalized model checking. In Computer Aided Verification (CAV), LNCS, Copenhagen, Denmark. 2002.

[16] C. E. Izard. Basic Emotions, Relations Among Emotions, and Emotion-Cognition Relations. Psychological Review. Vol. 99, No. 3, 561-565. 1992
[17] D. Aerts. Quantum Structure in Cognition. Journal of Mathematical Psychology, 53, 314-348, 2009.